



MEDICAL & DENTAL ADMISSION PROGRAM-2020

HIGHER MATH

Lecture : M-01

Chapter 05 : Permutation and Combination

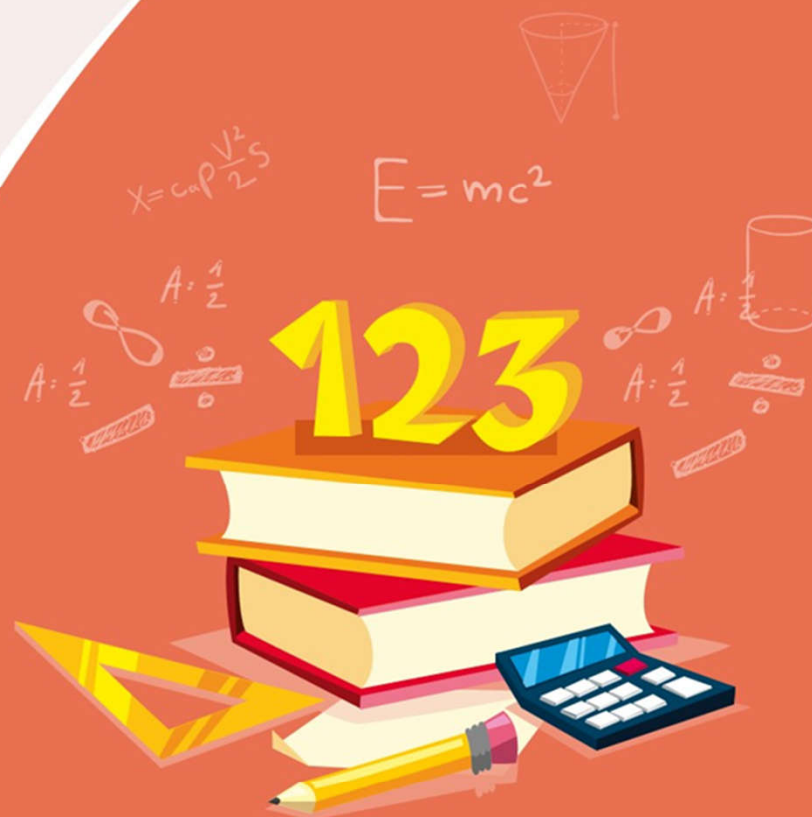


$$x = \frac{\sqrt{c^2 + c - \frac{b}{2}}}{c}$$



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Introduction

abc

ab ↔ ba
bc ↔ cb
ca ↔ ac

0
1
2
3

7P2 = 7 × 6
10P3 = 10 × 9 × 8

$${}^{12}C_3 = \frac{12 \times 11 \times 10}{3!} = \frac{12 \times 11 \times 10}{3 \times 2 \times 1} = 365$$

$$365C_2 = \frac{365 \times 364}{2 \times 1}$$

$${}^nP_r = {}^nC_r \times r!$$

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$= \frac{n(n-1)(n-2) \dots (n-r+1) \{n-r\}!}{(n-r)!}$$

$$= n(n-1)(n-2) \dots (n-r+1)$$

no. factors = r

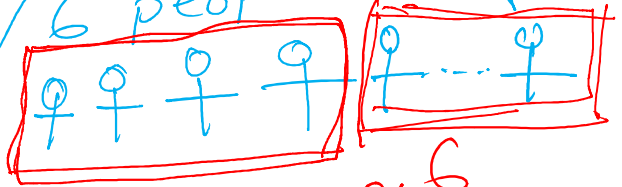
r (Diff.) → r!

1 → 10 Arr.
5 → 5 × 10 Comb.

$$n! = n(n-1)(n-2) \dots 3 \times 2 \times 1$$

$$= n \{ (n-1)! \} = n(n-1) \{ (n-2)! \}$$

6 people among 10



$${}^n C_x = {}^n C_y \text{ iff } x = y \text{ or } x + y = n$$

Poll Question 01

If ${}^{15}C_3 = {}^{15}C_x$ find value of x-

- (a) 12
- (b) 3
- ✓ (c) Both a & b
- (d) None

$${}^{15}C_3 = {}^{15}C_x$$

$$x = 3$$

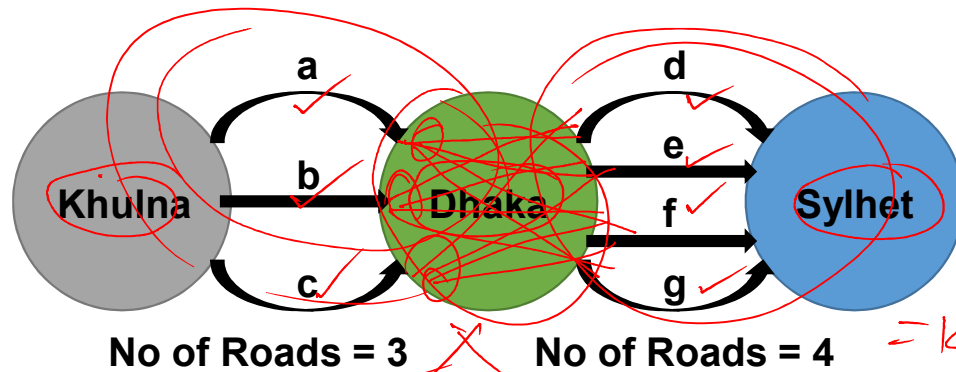
$$3 + x = 15$$
$$\Rightarrow x = 12$$

If ${}^nC_x = {}^nC_y$ then,

$$\boxed{x = y} \text{ or } \boxed{x + y = n}$$

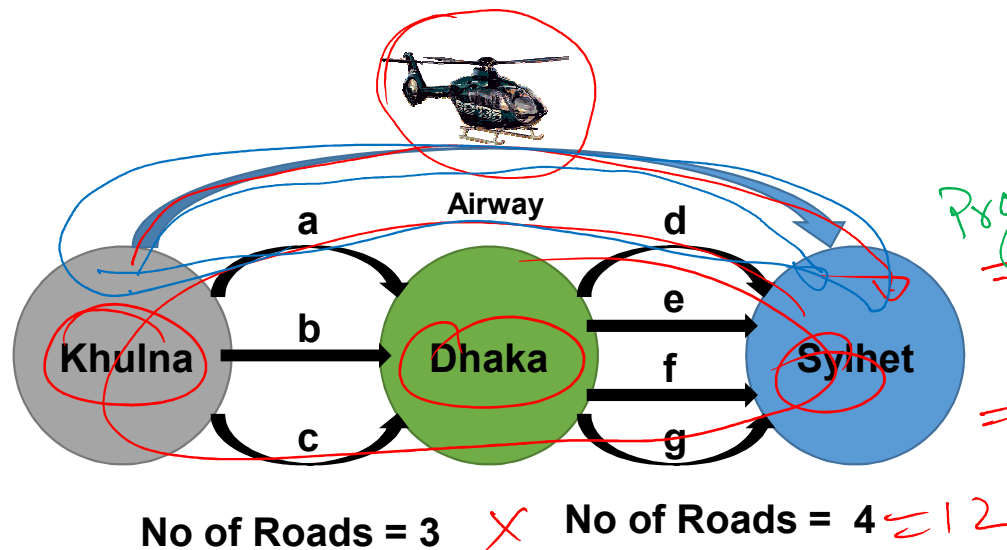
Fundamentals of Permutation

Rule of Product:



UNMESH
 $6 \times 5 \times 4 \times 3 \times 2 \times 1$
 $= 6!$

Rule of Sum:



Product
 $= 3 \times 4 + 1$
 $= 12 + 1$
 Sum

Permutation of objects when all objects are not unique or different or distinct

Ⓐ Number of ways in which n things can be arranged taking them all at a time, when p of the things are exactly alike of 1st type, q of them are exactly alike of a 2nd type, r of them are exactly alike of a 3rd type and the rest all are distinct is: $\frac{n!}{p! \times q! \times r!}$

How many arrangements can be made out of the letters of the word ENGINEERING?

$$\frac{11!}{3! 3! 2! 2! 1!}$$

WRONG!!

$$\frac{11!}{4! 3! 2! 2! 1!}$$

ENG R

Some caste issues

Taking the letters of the word 'DIRECTOR'-

- How many words can be formed?
- How many new words can be formed?
- How many new words can be formed where consonant occupy first position?
- How many new words can be formed where vowel occupy first position?
- How many words can be formed where all vowels remain (come) together?
- How many words can be formed where all vowels don't remain (come) together?
- How many words can be formed where any two vowels don't remain (come) together?
- How many words can be formed where positions of vowels remain unchanged?
- How many words can be formed where vowels and consonants don't change their relative position?
- How many words can be formed where positions of vowels don't change their order?
- How many words can be formed where 'R' occupy last place?

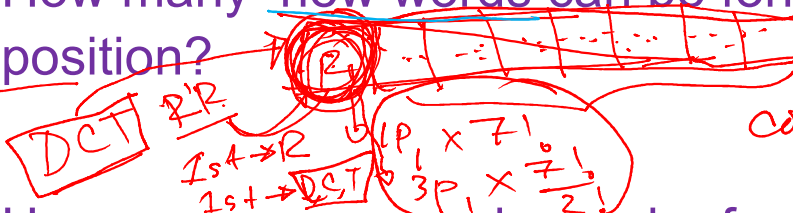
Some caste issues

Taking the letters of the word **DIRECTOR**-

- How many words can be formed?
- How many new words can be formed?

$$\frac{8!}{2!}$$

- How many new words can be formed where consonant occupy first position?

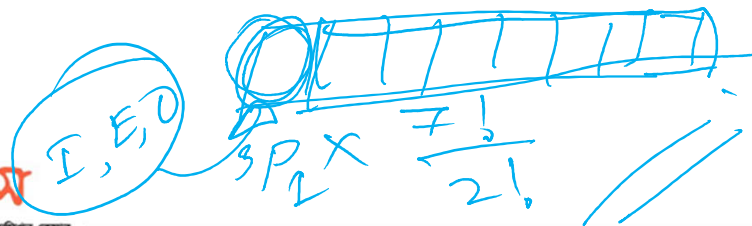


cons. occupy at 1st

$$= 7! + 3P_1 \times \frac{7!}{2!}$$

Ans: $7! + 3P_1 \times \frac{7!}{2!} - 1$

- How many new words can be formed where vowel occupy first position?



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Some caste issues

Taking the letters of the word 'DIRECTOR'-

- How many words can be formed where all vowels remain (come) together?

(I, E, O), D, R, C, T, R

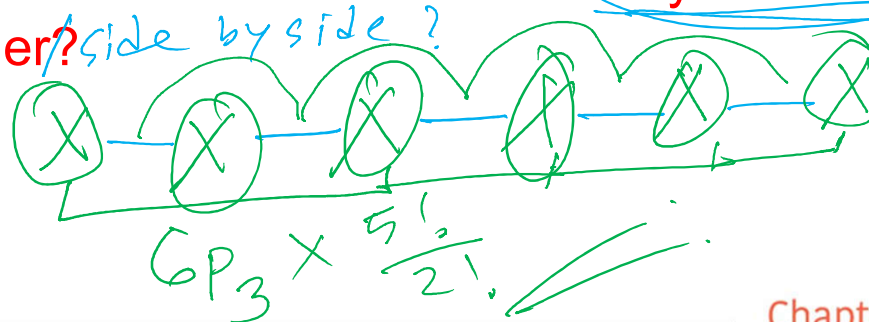
$$\frac{6!}{2!} \times 3!$$

- How many words can be formed where all vowels don't remain (come) together?

$$\frac{8!}{2!} - \frac{6!}{2!} \times 3!$$

DIRECTOR

- How many words can be formed where any two vowels don't remain (come) together?



Poll Question 02

✎ In how many ways the letters of the word 'KACHUA' can be arranged such that all vowels come together?

(a) 24

☒ (b) 72

(c) 144

(d) 8

$$\begin{array}{l} \text{[A, U, A]}, K, C, H \\ 4! \times \frac{3!}{2!} \\ = 24 \times 3 = 72 \end{array}$$

Some caste issues

Taking the letters of the word 'DIRECTOR'-

- How many words can be formed where positions of vowels remain unchanged?

$$\frac{5!}{2!}$$

- ✓✓ How many words can be formed where vowels and consonants don't change their relative position?

$$3! \times \frac{5!}{2!}$$

~~'DIRECTOR'~~

- $$\begin{array}{r} 8! \\ \hline 2! \cdot 3! \cdot 0! \end{array}$$

- Handwritten notes in red ink:
- A diagram of a cylinder with vertical lines and a circle labeled 'P'.
 - The text $7! \times P$ with a large 'X' over it.
 - A large circle containing the text $7!$.
 - The text $7! \times P$ with a large 'X' over it.

Permutation with repetition

❖ Number of permutations of n distinct things taking r at a time when each thing may be repeated any number of times is n^r

- In how many different ways 5 member can vote 3 person?

$V = a, b, c, d, e$

can 1. $\rightarrow 1, 2, 3$

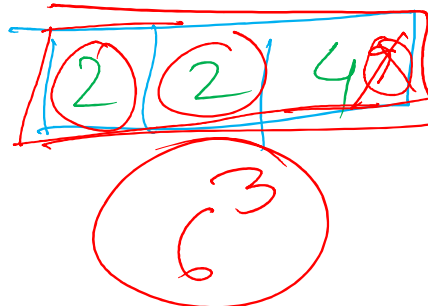
1	2	3
a	a	a
b	b	b
c	c	c
d	d	d
e	e	e

a	b	c	d	e
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3

$$3 \times 3 \times 3 \times 3 \times 3 = 3^5$$

- How many 3 digit numbers can be formed from digits 1, 2, 4, 6, 8, 9 with repetition?

$$6 \times 6 \times 6 = 6^3$$



Digits

Poll Question 03

A Grameenphone number starts with 017 or 013.

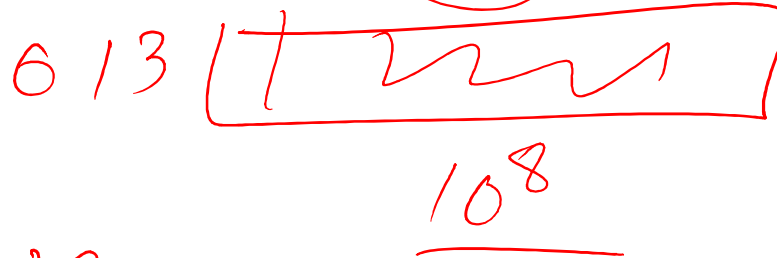
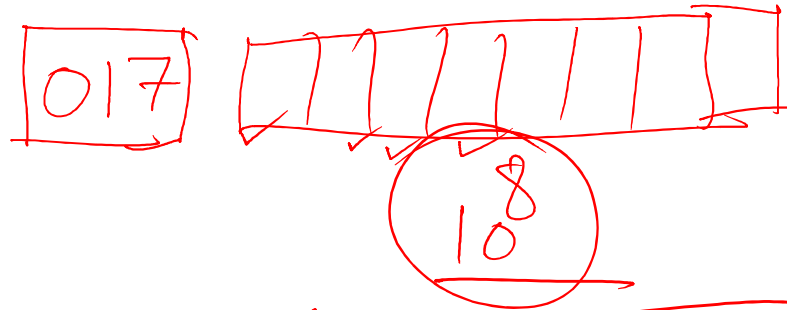
How many connections can Grameenphone provide?

(a) $^{10}P_8$

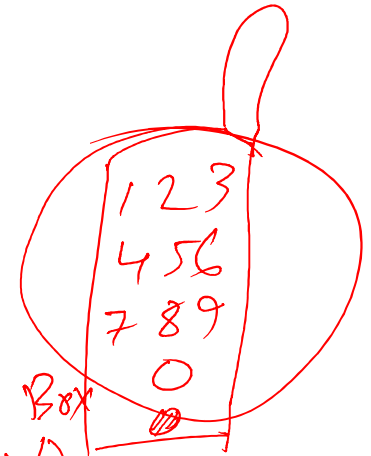
(b) 10^8

(c) $^{10}C_8$

✓ (d) 2×10^8



$$a + a = 2a$$



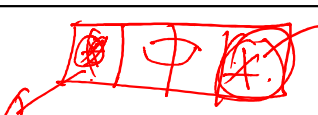
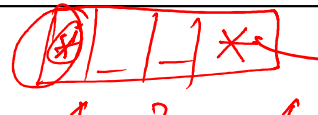


(Digit)

$$10^8 + 10^8 = 2 \times 10^8$$

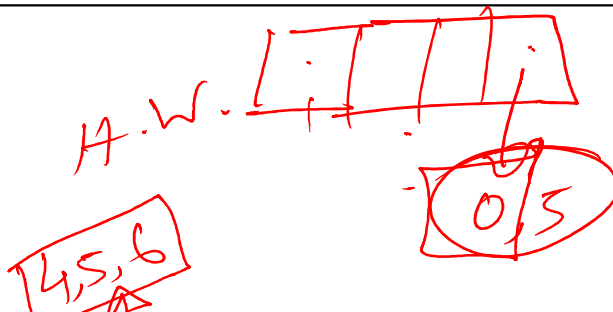
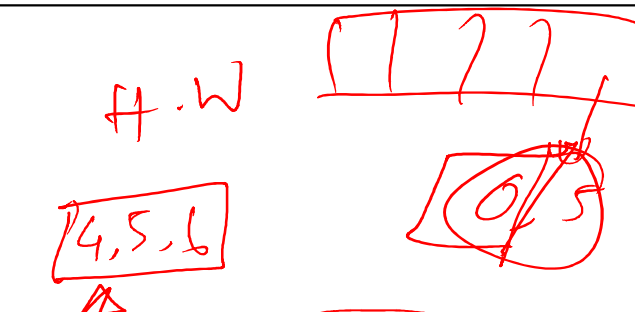
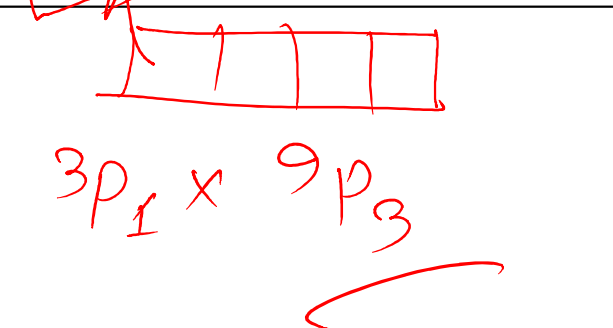
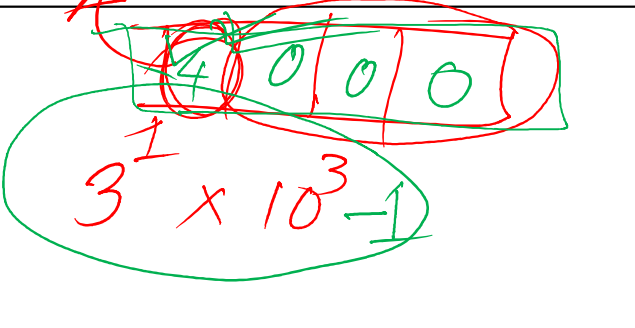
Related To Formation Of Numbers

From the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 using a digit- (i) once, (ii) multiple times-

Condition/Question	i) Using once (${}^n P_r$)	ii) Using multiple times (n^r)
(a) (a) How many <u>4 digit</u> numbers can be formed?	 ${}^9 P_1 \times {}^9 P_3$ ✓	 $9^1 \times 10^3$
(b) (b) How many <u>odd 4 digit</u> numbers can be formed?	 ${}^8 P_1 \times {}^8 P_2 \times {}^5 P_1$ ✓	 ${}^1 P_1 \times {}^2 P_2 \times {}^5 P_1$
(c) (c) How many <u>even 4 digit</u> numbers can be formed?	H.W.:	H.W.

Related To Formation Of Numbers

From the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 using a digit- (i) once, (ii) multiple times-

Condition/Question	i) Using once	ii) Using multiple times
(d) How many 4 digit numbers can be formed which are divisible by 5?	<p>H.W. </p>	<p>H.W. </p>
(e) How many numbers can be formed which are <u>greater than 4,000</u> but less than <u>7,000</u> ?	<p></p> <p>$3P_1 \times 9P_3$</p>	<p></p> <p>$3^1 \times 10^3 - 1$</p>

Circular permutations:

- Number of circular permutations (arrangements) of n distinct things = $(n - 1)!$ [when it will be observed by only one side]
- Number of circular permutations (arrangements) of n distinct things, when clockwise and anticlockwise arrangements are not different (i.e., when observations can be made from both sides) = $\frac{(n-1)!}{2}$ [when it will be observed by 2 sides]

- In how many ways 13 person can be seated around a round table?

$$(13-1)!$$

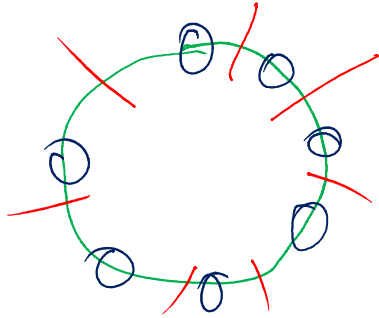


- How many chain can be made using 10 stones of distinct color or shape?

$$\rightarrow \bigcirc \leftarrow \frac{(10-1)!}{2}$$

Math related to circular table

✎ In how many ways 7 students of science group and 5 students of arts group can be seated around a circular table such that none of the students of arts group sit together?



$$(7-1)! \times$$

2020. 2020. 2020. Perm.
2020. Cir. Perm.

7P5

Gen. Perm.



Form group on several condition

01. In how many ways a group of 15 people can be made choosing from 40 people?

Case – 1: If Ashik is in the group:

$${}^{39}C_{14}$$

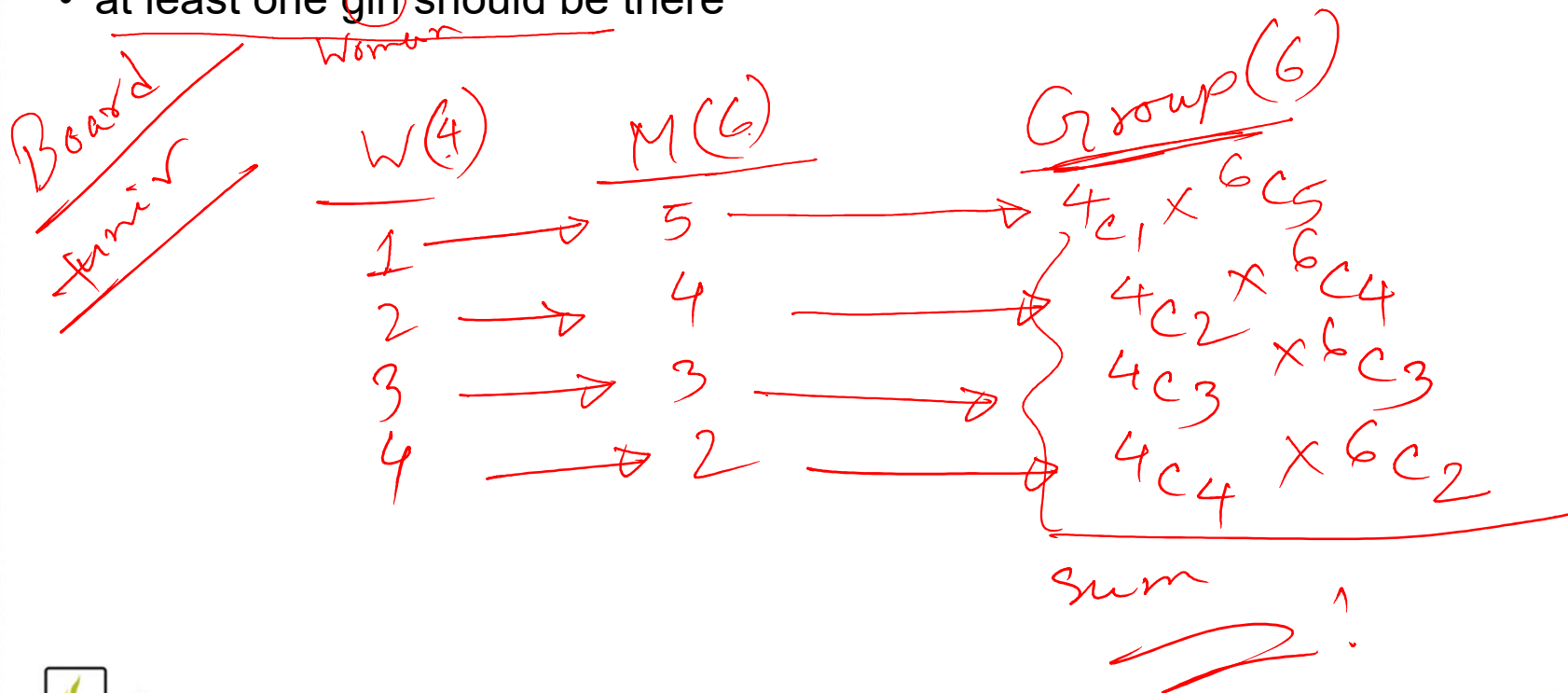
Case – 2: If Ashik is not in the group:

$${}^{39}C_{15}$$

Form group on several condition

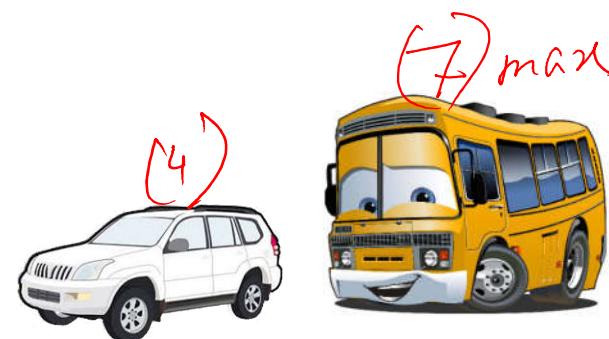
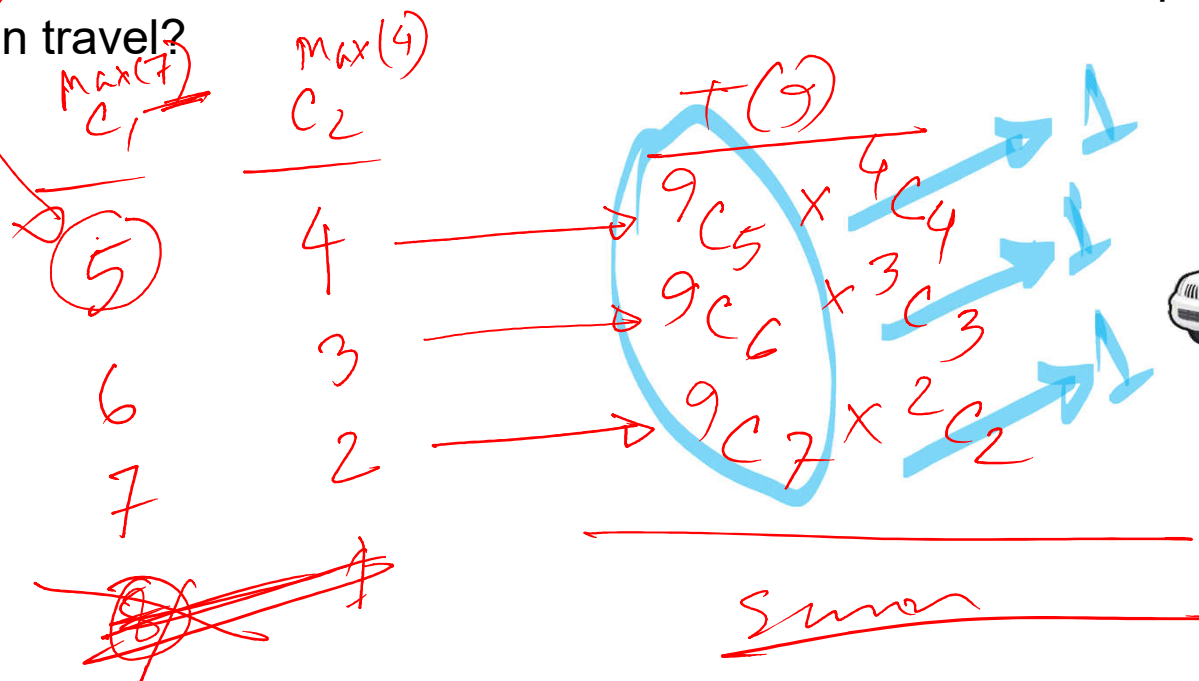
From a group of 10 person including 4 women, 6 persons are to be selected. In how many ways can it be done? - such that:

- at least one girl should be there



Form group on several condition

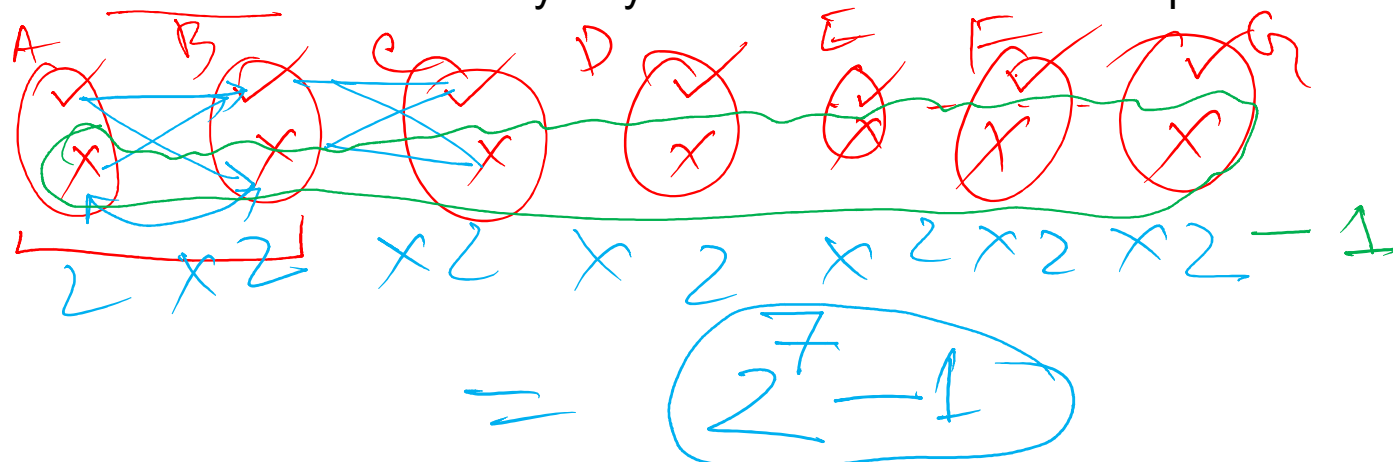
A group of 9 persons can travel by two car, one car cannot accommodate more than 7 persons and the another cannot accommodate more than 4 persons. In how many ways the can travel?



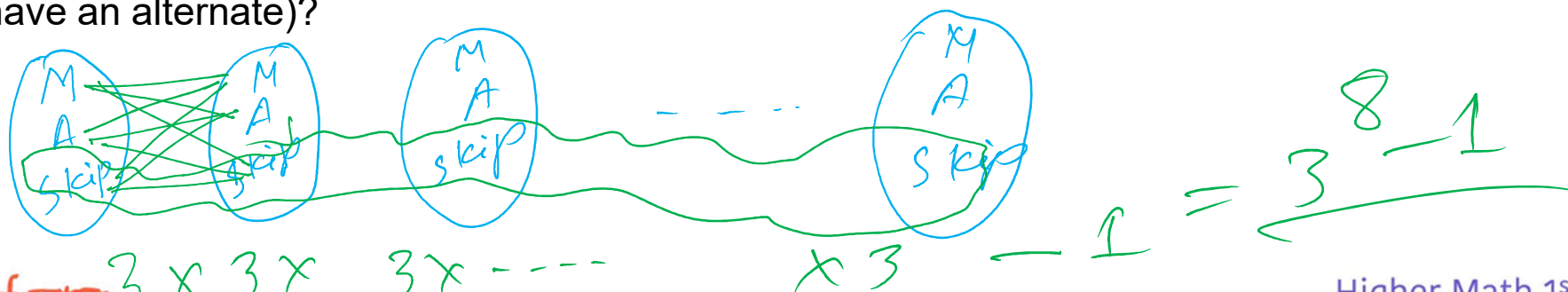
$$nC_n = 1$$

Related To The Selection Of One Or Multiple Objects

01. One has 7 friends. In how many ways he can invite one or multiple friends of him?



02. How many ways are there to choose one or multiple questions from 8 questions (each question have an alternate)?



Geometry related problem.

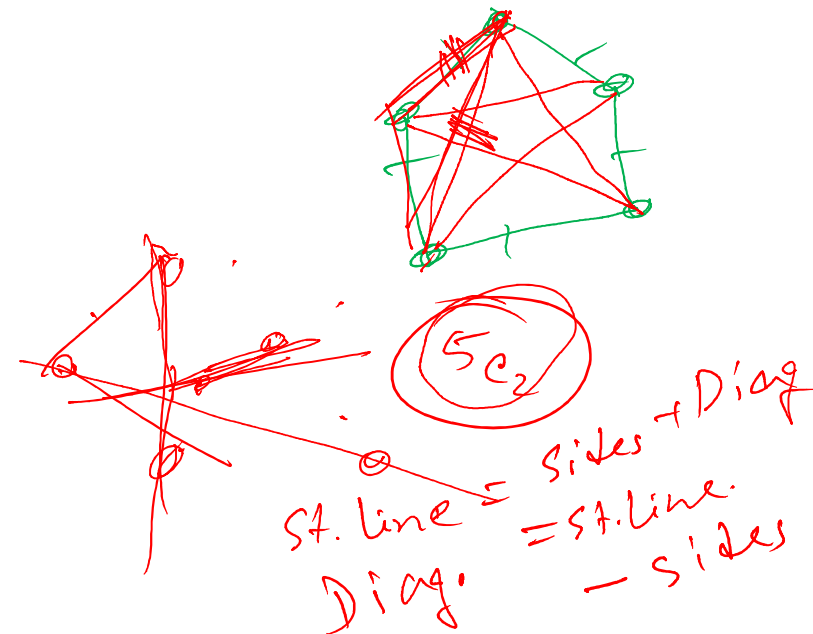
• How many _____ can be formed using the vertex points of a polygon of 10 arms?
straight line = $^{10}C_2$

Arm = 10

Diagonal = $^{10}C_2 \rightarrow 10$

Triangle = $^{10}C_3$

Quadrilateral = H.W.



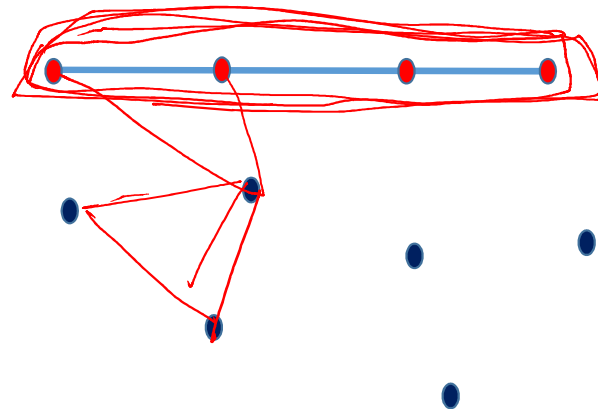
Geometry related problem.

- Find the number of triangles, straight lines that can be formed using 10 points in a plane such that 4 points are collinear?

Tri^o

$$10C_3 - 4C_3$$

St. Lines! $10C_2 - 4C_2 + 1$



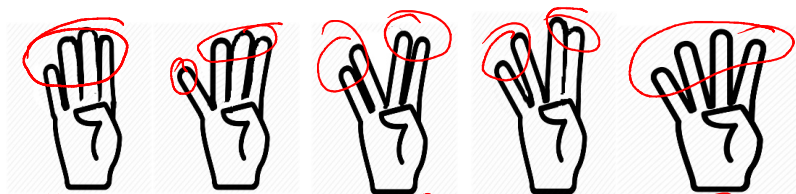
Related To The Combined Problems Of Permutation & Combination

Find the no. of selection that can be formed taking 4 letters from the word "ENGINEERING".

Ways	Examples	Number of Combinations
$1I + 1S$	VEEER ✓	$2C_1 \times 4C_1$
$2D$	EEGG	$4C_2$
$1D + 2S$	EEGR	$4C_1 \times 4C_2$
$4S$	ENGR	$5C_4$

Sum

~~(EEE) (NNN) (GG) (II) (R)~~



T → 2
D → 4
S → 5

Poll Question 04

Find the different number of selection taking 3 digit each time from the word 'SCHOOL' - ^{letters}

(a) 10

(b) 14

(c) 4

(d) 15

~~(S)(C)(H)(O)(L)~~
D → 1
S → 5

X

1D + 1S

$$\rightarrow 1 \times 4C_1 = 1 \times 4 = 4$$

3S

$$\rightarrow 5C_3 = \frac{5 \times 4 \times 3}{3 \times 2 \times 1} = 10$$

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ধ্বংস করে

$$X = c \rho \frac{V^2}{2} S$$

$$X = c \rho \frac{V^2}{2} S$$

$$E = mc^2$$

$$x = \sqrt{\frac{c^2}{c}} + c - \frac{b}{2}$$



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