MEDICAL & DENTAL ADMISSION PROGRAM-2020

HIGHER MATH

Lecture   :   M-03

Chapter 03   :   Straight Line

V.V.I.

(i) Converting co-ordinate of a point from polar to Cartesian:

(r, θ) → known

(x, y) → unknown

Example: If polar coordinate of a point is $(3, \ 90°)$, then what is its Cartesian coordinate?

[JU'14-15]

Soln:

$x = 3 \cos 90° = 3 \times 0 = 0$

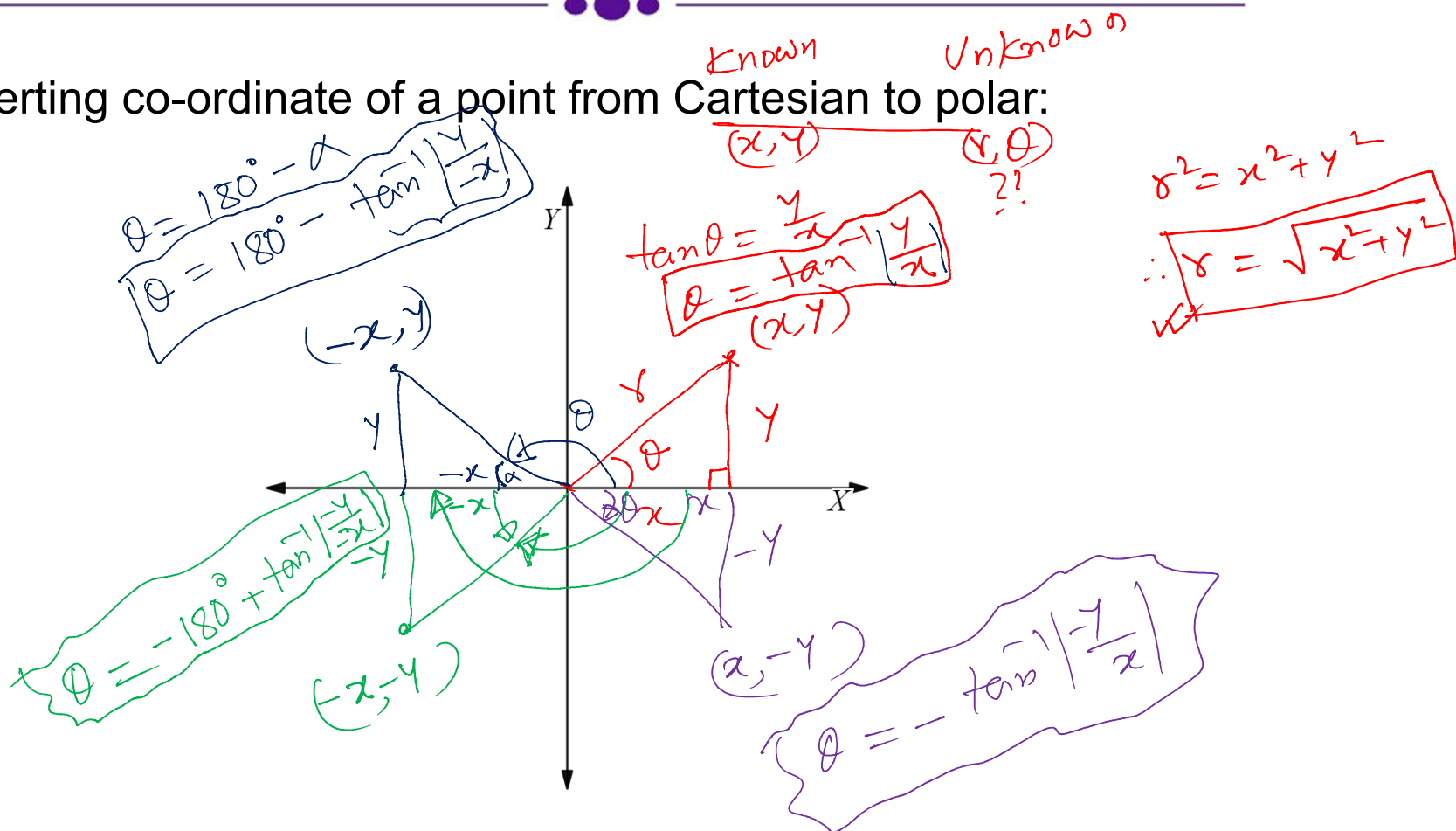$y = 3 \sin 90° = 3 \times 1 = 3$

$c(0, 3)$ Ans.

$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$

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(ii) Converting co-ordinate of a point from Cartesian to polar:

Known   Unknown θ

$(x, y) \longrightarrow (r, \theta)$?!

$$\theta = 180° - \alpha$$
$$\theta = 180° - \tan^{-1}\left|\frac{y}{x}\right|$$

$(-x, y)$

$$\tan\theta = \frac{y}{x}$$
$$\theta = \tan^{-1}\left|\frac{y}{x}\right|$$
(x, y)

$$r^2 = x^2 + y^2$$
$$\therefore r = \sqrt{x^2 + y^2}$$

$$\theta = -180° + \tan^{-1}\left|\frac{-y}{-x}\right|$$
$(-x, -y)$

$(x, -y)$
$$\theta = -\tan^{-1}\left|\frac{-y}{x}\right|$$
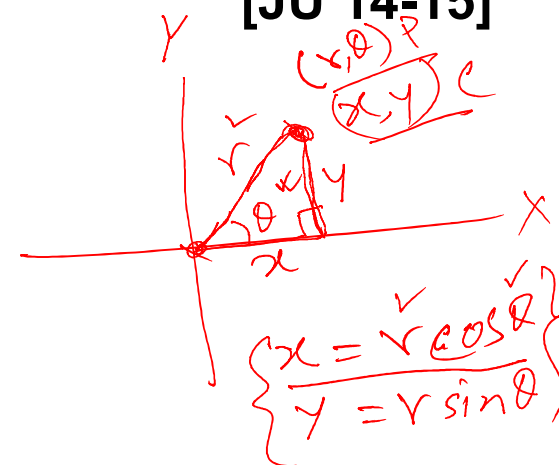
Type-1: Change of Coordinate System

● ● ●

(ii) Converting co-ordinate of a point from Cartesian to polar:

Example: If Cartesian coordinate of a point is $(-1, \sqrt{3})$, then what is its polar coordinate?

[JU' 18-19, 11-12, JnU' 12-13]

(r, θ)

$2?$

$(-1, \sqrt{3})$

$$r = \sqrt{(-1)^2 + (\sqrt{3})^2}$$
$$= \sqrt{4}$$
$$= 2$$

$(r, \theta) = (2, 120°)$

Ans.

$$\theta = 180° - \tan^{-1}\left|\frac{\sqrt{3}}{-1}\right|$$
$$= 180° - \tan^{-1}\sqrt{3}$$
$$= 180° - 60° = 120° = \frac{2\pi}{3}$$
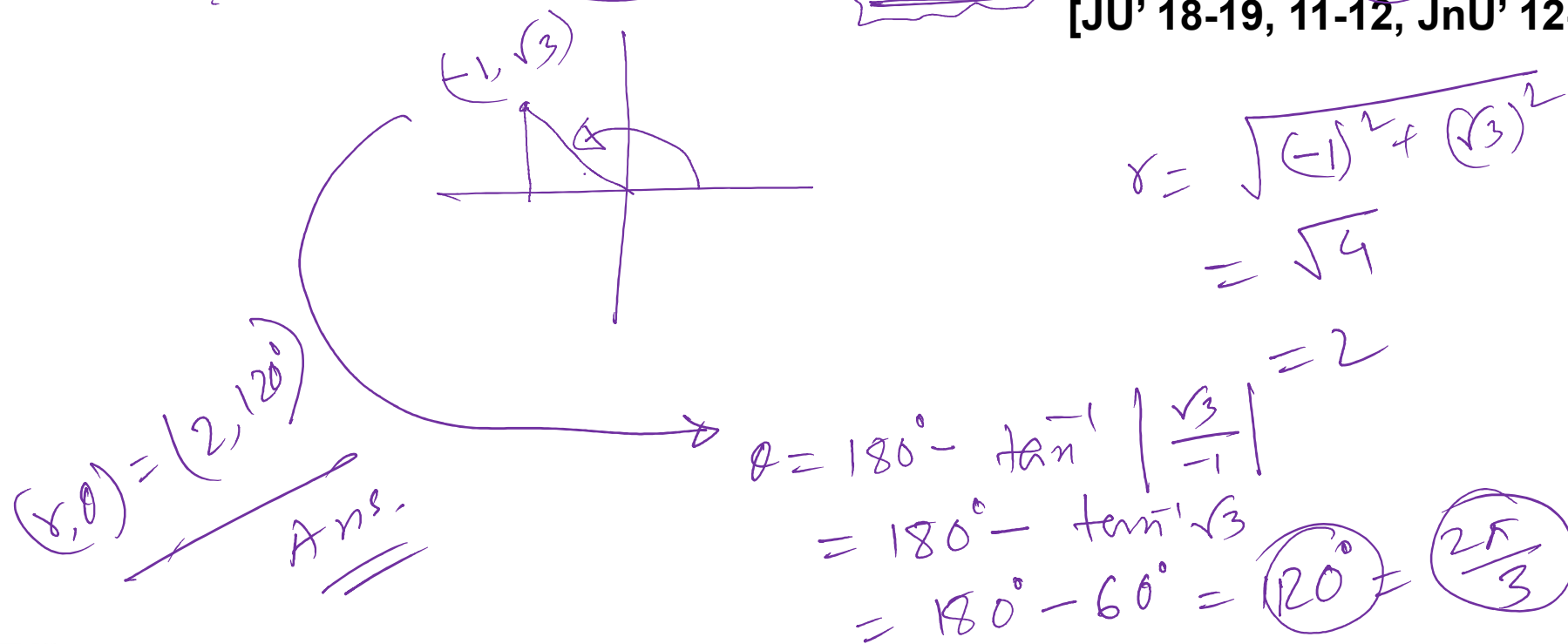
Type-1: Change of Coordinate System

(iii) Converting Cartesian equation to polar equation:

➤ Replace x by $r\cos\theta$ and replace y by $r\sin\theta$

➤ Change $x^2 + y^2$ with r^2

Example: Transform from Cartesian equation to polar equation: $x^2 + y^2 - 2ax = 0$ **[CU'14-15]**

Sol^n!

$$r^2 - 2ar\cos\theta = 0$$

$$\Rightarrow r - 2a\cos\theta = 0$$

$$\Rightarrow \boxed{r = 2a\cos\theta}$$

Ans

Type-1: Change of Coordinate System

(iv) Converting polar equation with Cartesian equation:

➢ Change r^2 with $x^2 + y^2$

➢ Replace $r\cos\theta$ and $r\sin\theta$ with x and y respectively.

Example: Transform from polar equation to Cartesian equation: $r = a\sin\theta$

$$\text{Sol}^n:$$

$$r = a\sin\theta$$

$$\Rightarrow r \cdot r = a r \sin\theta$$

$$\Rightarrow r^2 = a y$$

$$\Rightarrow x^2 + y^2 = a y \qquad \boxed{Ans}$$

Transform from polar equation to Cartesian equation: $2r\sin^2\left(\frac{\theta}{2}\right) = 1$ **[DU'18-19]**

(a) $x^2 + y^2 = (1 + x)^2$ ✓

(b) $y^2 = 1 + 2x$ ✓

(c) both a & b

(d) None

$2\sin^2\frac{\theta}{2} = 1 - \cos\theta$

$2r\sin^2\frac{\theta}{2} = 1$

$\Rightarrow r(1 - \cos\theta) = 1$

$\Rightarrow r - r\cos\theta = 1$

$\Rightarrow r - x = 1$

$\Rightarrow (r)^2 = (1+x)^2$
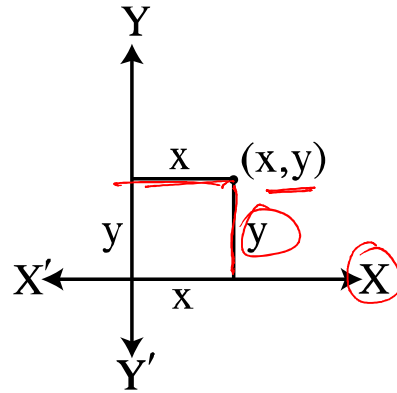
$\Rightarrow x^2 + y^2 = (1+x)^2 \to$ (a)

$\Rightarrow x^2 + y^2 = 1 + 2x + x^2$

$\Rightarrow y^2 = 1 + 2x \to$ (b)

Higher Math 1st Paper
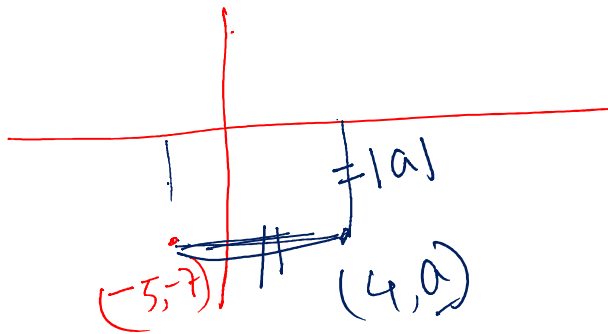Chapter 03 : Straight Line

Type-2: Problems related to determination of distance

➤ For (x, y) point ⎰ i) Distance from x-axis = $|y|$ units

⎱ ii) Distance from y-axis = $|x|$

➤ Distance between (x_1, y_1) and (x_2, y_2), d = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ units

Type-2: Problems related to determination of distance

Example: If distances of point $(4, a)$ from x-axis and point $(-5, -7)$ are equal, then what is the value of a?   **[JU'18-19, JU'11-12,RU'17-18]**

A/C,

$$|a| = \sqrt{(4+5)^2 + (a+7)^2}$$

$$\Rightarrow a^2 = 81 + a^2 + 14a + 49$$

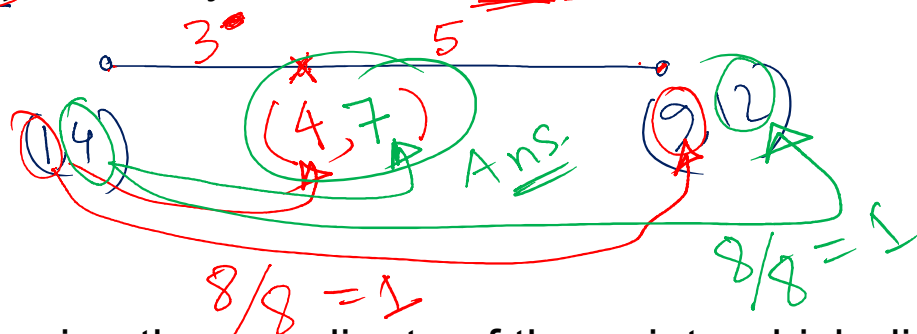$$\Rightarrow 14a = -130$$

$$\therefore a = -\frac{65}{7} \quad \text{Ans.}$$

Poll Question-02

If distances of point $(a, 5)$ from y-axis and point $(7,2)$ are equal, then what is the value of a?

(a) 19/7

(b) 29/7

(c) 19/9

(d) 29/9

$$A/c,\ |a| = \sqrt{(a-7)^2 + (5-2)^2}$$

$$\Rightarrow x^2 = x^2 - 14a + 49 + 9$$
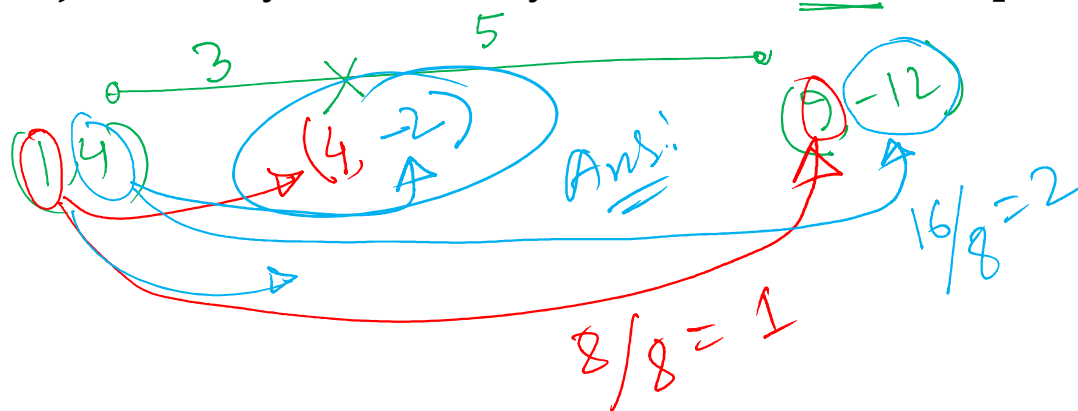
$$\Rightarrow 14a = 58$$

$$\therefore a = 29/7$$

Example: Determine the coordinate of the points which divide the line connecting points $(1,4)$ and $(9, 12)$ internally in the ratio $3:5$ **[DU'05-06,14-15]**
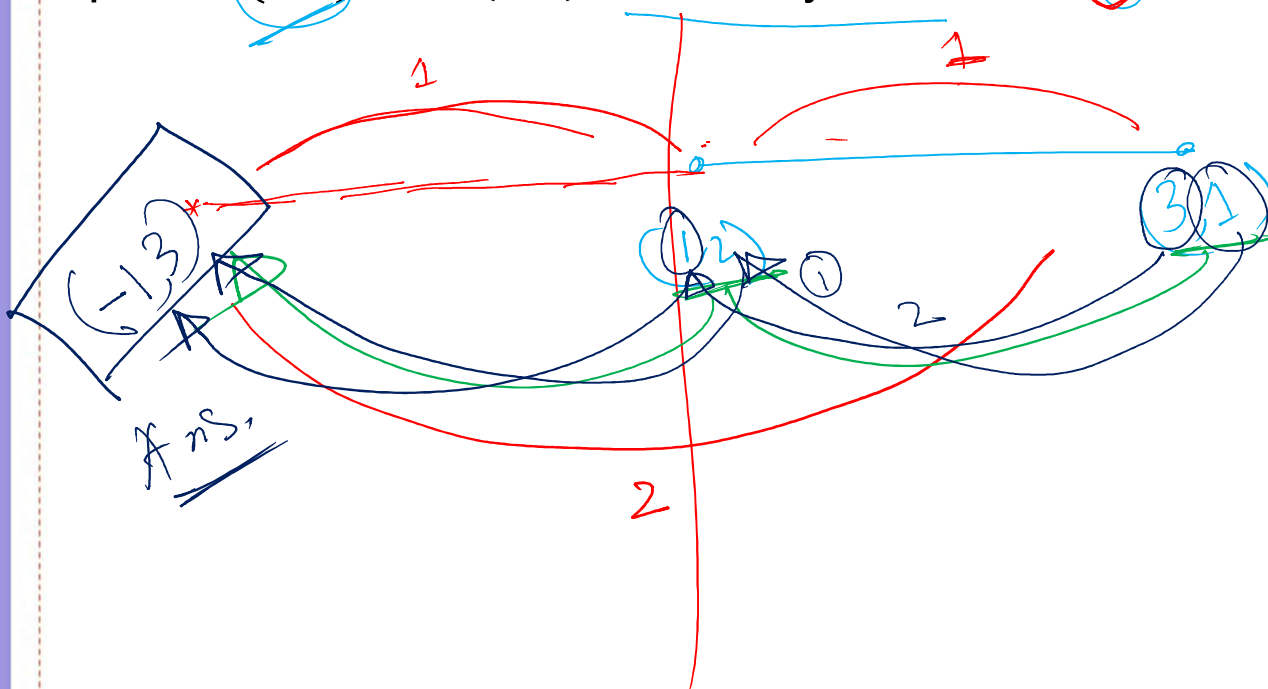

Example: Determine the coordinate of the points which divide the line connecting points $(1,4)$ and $(9, -12)$ internally & externally in the ratio $3:5$. **[DU' 05-06, 14-15, JU' 18-19]**

Example: Determine the coordinate of the points which divide the line connecting points (1,2) and (3,1) externally in the ratio 1:2.

Example: In what ratio x and y axis divide the segment connecting the points (3,2) and (5,-7) ?

x-axis divides the st. line segment; connecting the

points (x_1, y_1) & $(x_2, y_2) = \dfrac{y_1}{y_2}$

$-ve \Rightarrow$ Int.

$+ve \Rightarrow$ Ext.

y-axis — — — — — — — — $= \dfrac{x_1}{x_2}$

Sol^n:

x-axis : $\dfrac{m}{n} = \dfrac{2}{-7}$ is $-ve$
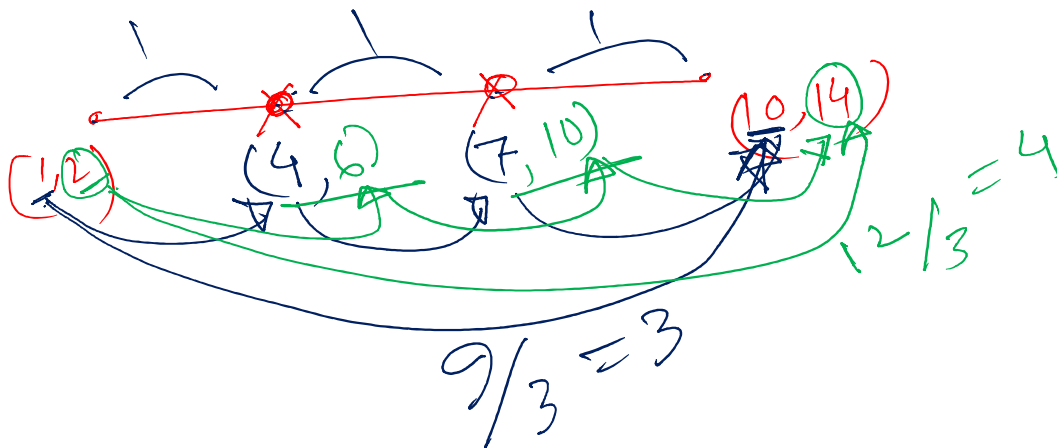
$\therefore m:n = 2:7$ (Int.)

y-axis: $\dfrac{p}{q} = \dfrac{3}{5}$ $\Rightarrow +ve$
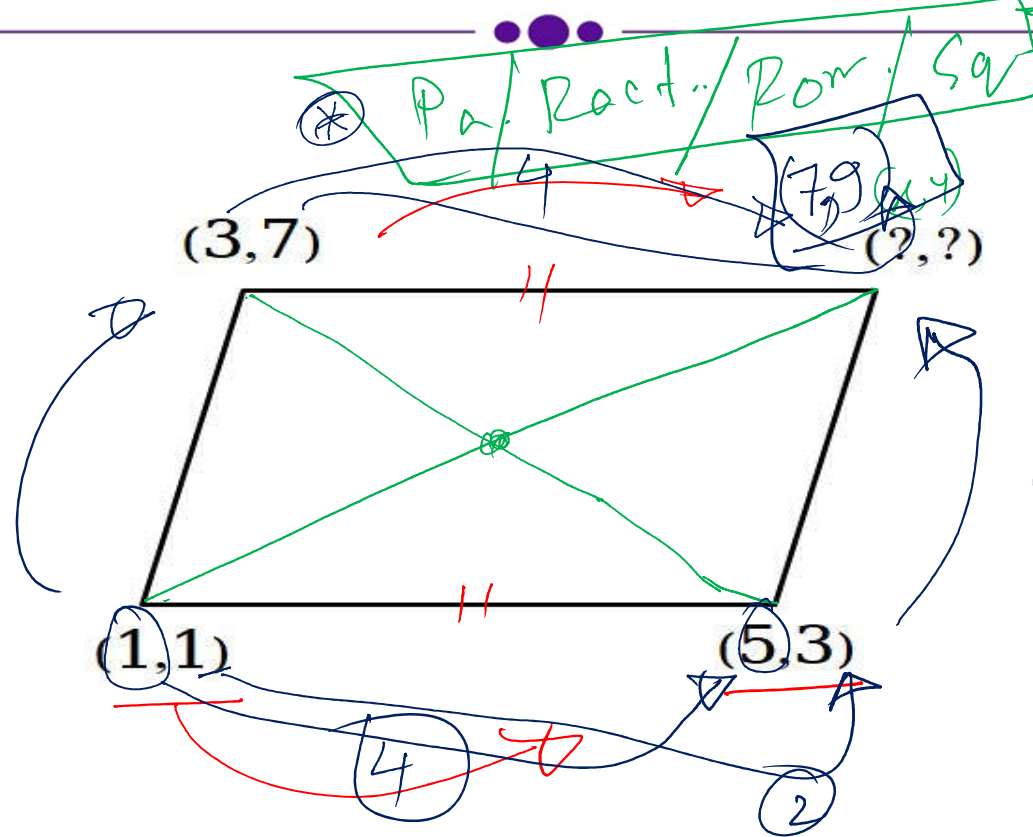
$\therefore p:q = 3:5$ (Ext)

Find the points of trisection of the segment connecting the points (1,2) and (10, 14).

(a) $(4, 6)$ & $(7,10)$

(b) $(6,4)$ & $(7,10)$

(c) $(4, 6)$ & $(10,7)$

(d) $(6, 4)$ & $(10,7)$

Example:

Pa. / Rect.. / Rom. / Sq

(7,9) (x,y)

(3,7)   (?,?)

(1,1)   (5,3)

written!

$$\frac{x+1}{2} = \frac{3+5}{2}$$

$$\Rightarrow x = 3+5-1$$
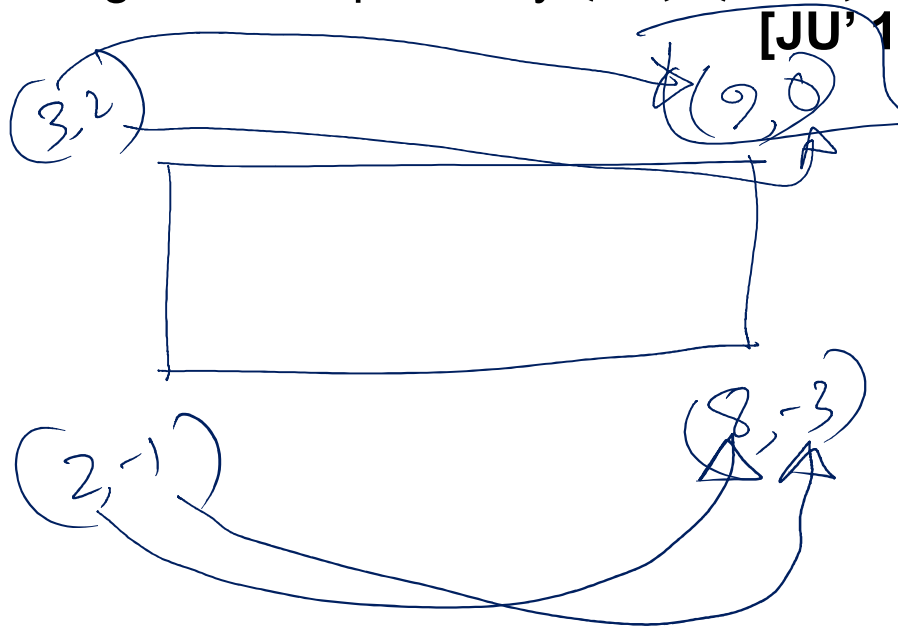
$$\frac{y+1}{2} = \frac{7+3}{2}$$

$$\Rightarrow y = 7+3-1$$

The three vertices of a rectangle are respectively $(3,2), (2,-1)$ & $(8,-3)$. Find its fourth vertex. **[JU' 10-11, KU' 06-07]**

(a) (0,9)

(b) (9,0)

(c) (0,-9)

(d) (-9,0)

Type-5 Area related problem

Example: The vertices of triangle are $(-1, -2)$, $(2, 5)$ and $(3, 10)$; determine the area of the triangle.   **[DU' 14-15, 01-02, 03-04, JU' 18-19, 17-18, 11-12, CU' 12-13]**
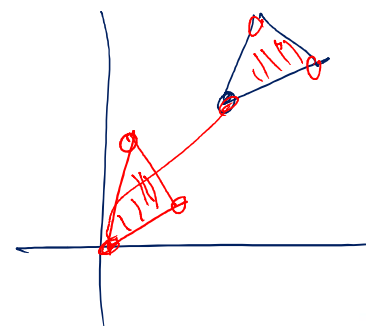
$(-1, -2)$ $(2, 5)$ $(3, 10)$

$(0, 0)$ $(3, 7)$ $(4, 12)$

$$A = \frac{1}{2}\left| 3 \times 12 - 7 \times 4 \right| \text{ sq. units}$$

$$= \frac{1}{2}\left| 36 - 28 \right|$$

$$= 4 \text{ sq units}$$

$$A = \frac{1}{2}\left| \begin{matrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{matrix} \right| \text{ sq unit}$$

(x_1, y_1)
(x_3, y_3)
(x_2, y_2)

Type-6: Determination of coordinates of different points of triangle

In the triangle ABC formed by $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$,centroid $= \left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3} \right)$

Example: The three vertices of a triangle are respectively (0,0), (0,3) & (4,0). Find its center of mass. **[BAU' 14-15]**
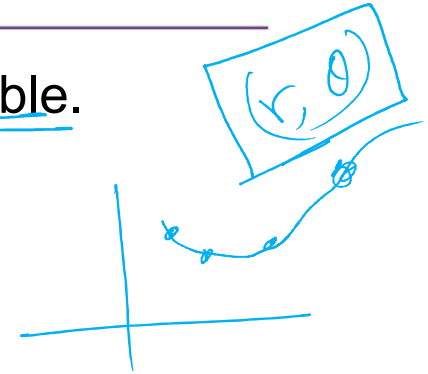
$$C = \left(\frac{0 + 0 + 4}{3}, \quad \frac{0 + 3 + 0}{3} \right)$$

$$= \left(\frac{4}{3}, \quad 1 \right) \quad Ans$$

Type-7: Determination of equation of locus

Example: Find the locus of $(2t + 1, 3t - 1)$ where t is variable.

(r, θ)

$$\left\{ \begin{array}{l} x = 2t + 1 \\ y = 3t - 1 \end{array} \right\} \text{Para. eq}^n$$

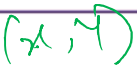$$\boxed{\frac{x-1}{2} = \frac{y+1}{3}} \rightarrow \text{st. line}$$

Example: Find the locus of $(a\cos\theta, a\sin\theta)$. $\rightarrow$ θ variable

$$\left\{ \begin{array}{l} x = a\cos\theta \\ y = a\sin\theta \end{array} \right\}$$

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{a}\right)^2 = 1$$

$$\Rightarrow x^2 + y^2 = a^2 \quad \text{eq}^n \text{ of circle.}$$

(x, y)

The distance of a set of points from the x axis is always half its distance from the origin. Find the equation of the locus of such a set of points. **[JU' 09-10]**

(a) $x^2 + y^2 = 4y^2$

(b) $x^2 = 3y^2$

(c) both a & b

(d) none

$$|y| = \frac{1}{2}\sqrt{(x-0)^2 + (y-0)^2}$$

$$\Rightarrow y^2 = \frac{1}{4}\left(x^2 + y^2\right)$$

$$\Rightarrow x^2 + y^2 = 4y^2$$

$$\Rightarrow x^2 = 3y^2$$

Type-8: Finding the slop

➢ Slope of the line formed by joining (x_1, y_1) and (x_2, y_2), $\boxed{m = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{\text{Difference of the ordinates}}{\text{Difference of the abscissas}}}$

➢ If a line segment form angle θ with positive side of x-axis, then slope $m = \tan\theta$

➢ If $\boxed{ax + by + c = 0}$ is the equation of a straight line, then slope, $m = \dfrac{-a}{b} = -\dfrac{\text{coefficient of } x}{\text{coefficient of } y}$

Example: What is the angle Produced by $x - y + 4 = 0$ with the y axis?   **[DU' 18-19]**
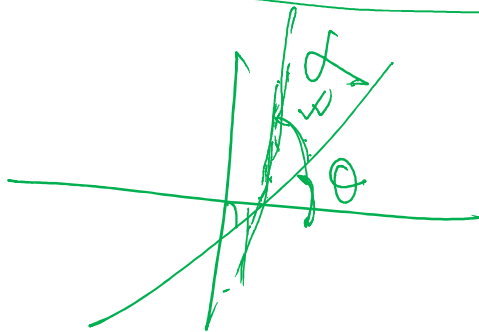
$y = \textcircled{m}x + c$

$x - y + 4 = 0$

$\Rightarrow y = x + 4$

$\text{Slope} = 1$

$\tan\theta = 1$

$\boxed{\theta = 45°}$

$\alpha = 90° - \theta$

$= 90° - 45°$

$= 45°$

Type-9: Determination of equation of straight line on different conditions

Case-01: Slope & intercept from y axis are given

A straight line has a slope of m and intercept at y-axis is c, then it's equation is $\Rightarrow$ $\boxed{y = mx + c}$

Example: The slope of a straight line is 2 & it intercepts 5 unit from positive direction of y axis. Find the equation of the line.

$$m = 2 \qquad c = 5$$

$$y = 2x + 5 \qquad \rightarrow Ans.$$

Case-02: If point & slope are given.

A straight line has a slope of m and it passes through point (x_1, y_1), then it's equation is ⇒

$$(y - y_1) = m(x - x_1)$$

Example: Find the equation of line passing through point (5, 2) and with slope 3.

$$y - 2 = 3(x - 5)$$

Ans.

Type-9: Determination of equation of straight line on different conditions

Case-03: Passing through two definite points.

A straight line passing through of (x_1, y_1) and (x_2, y_2), then equation of straight line

is $\Rightarrow \boxed{\dfrac{y-y_1}{y_1-y_2} = \dfrac{x-x_1}{x_1-x_2}}$

Example: Find the equation of straight line passing through points (3,4) & (5,2).

$$\frac{x-3}{3-5} = \frac{y-4}{4-2}$$

Ans.

Type-9: Determination of equation of straight line on different conditions

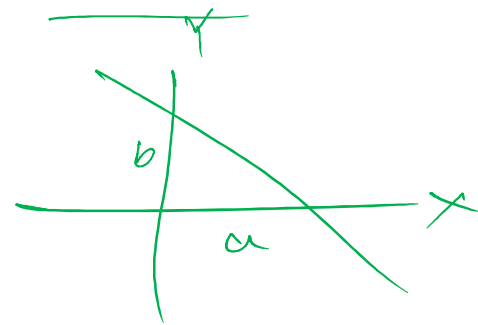**Case-04:** The intercept from both axes are given.

If a straight line has intercept of a and b respectively from x-axis and y-axis, then it's equation is $\Rightarrow \boxed{\dfrac{x}{a} + \dfrac{y}{b} = 1}$

Example: A straight line intercepts 4 & 3 unit respectively from positive directions of x & y axis. Find the equation of the line.

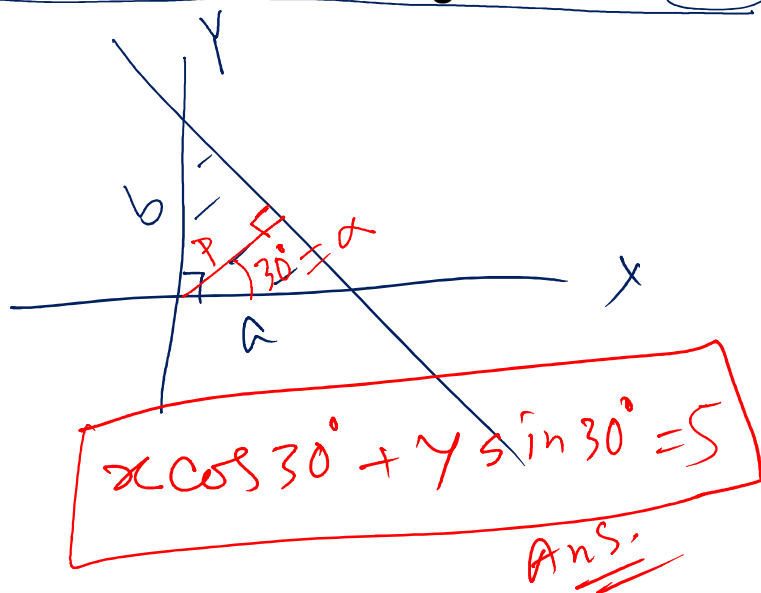$$a = 4 \qquad b = 3$$

$$\frac{x}{4} + \frac{y}{3} = 1 \qquad Ans.$$

Type-9: Determination of equation of straight line on different conditions

Case-05: Related to the formula $\boxed{x\cos\alpha + y\sin\alpha = p}$

$$\frac{x}{a} + \frac{y}{b} = 1$$

Equation of a line is $x\cos\alpha + y\sin\alpha = p$, where 'p' is the perpendicular distance from the origin to the line and α is the angle made by the perpendicular with positive x-axis.

$$\frac{x}{P/\cos\alpha} + \frac{y}{P/\sin\alpha} = 1$$

Example: A straight line forms a triangle of $\frac{50}{\sqrt{3}}$ sq. Unit with the axes and the perpendicular drawn to it from origin makes $30°$ angle with x-axis find the equation of the line   [RU'18-19]

$$\Delta = \frac{1}{2}ab$$

$$\frac{50}{\sqrt{3}} = \frac{1}{2} \cdot \frac{P}{\cos\alpha} \cdot \frac{P}{\sin\alpha}$$

$$\frac{50}{\sqrt{3}} = \frac{P^2}{\sin 2\alpha}$$

$$\Rightarrow \frac{50}{\sqrt{3}} = \frac{P^2}{\frac{\sqrt{3}}{2}}$$

$$\frac{P^2}{\sin 60°} \Rightarrow P^2 = 25$$

$$\therefore P = 5$$

$$\boxed{x\cos 30° + y\sin 30° = 5}$$

Ans.

Type-10: Equation of parallel line

Example: Find the equation of a line parallel to the line $3x + 4y + 5 = 0$ which passes through the point $(1, 2)$.

$$ax + by + c = 0$$

Parallel: $a\overset{a}{x} + by + k = 0$

$$3x + 4y \oplus k = 0$$

$(1,2) \rightarrow \boxed{3 \times 1 + 4 \times 2} + k = 0$

$$\Rightarrow k = -(3 \times 1 + 4 \times 2)$$

Sol^n:

$$3x + 4y \ominus (3 \times 1 + 4 \times 2) = 0$$

$k =$

For what value of α, $(\alpha - 1)x + (\alpha + 1)y - 7 = 0$ line will be parallel to $3x + 5y + 4 = 0$? **[RU'08-09]**

(a) 4

(b) -4

(c) 0

(d) 7

$\alpha - 1 = 3$

$\alpha + 1 = 5$

$\alpha = 4$

$\begin{cases} a_1 x + b_1 y + c_1 = 0 \\ a_2 x + b_2 y + c_2 = 0 \end{cases}$

Parallel : $\boxed{\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2}}$

Type-11: Equation of perpendicular line

Example: Find the equation of a line perpendicular to the line $3x + 4y + 5 = 0$ which passes through the point (3, 4).

$$ax + by + c = 0$$

Perpendicular $\rightarrow$ $bx - ay + k = 0$

Sol^n:

Perpendicular

$$4x - 3y - (4 \times 3 - 3 \times 4) = 0$$

$$\Rightarrow 4x - 3y = 0$$ Ans

Poll Question-07

For what value of k, $2x + 3y + 5 = 0$ line will be perpendicular to $3x + ky + 6 = 0$?

(a) 2

(b) 3

(c) -2

(d) -3

Type-12: Angle between two straight lines

$\bullet\; \bullet\; \bullet$

➤ The slope of two straight line is m_1 and m_2. If the angle between them is θ then, $\boxed{\tan\theta = \pm \dfrac{m_1 - m_2}{1 + m_1 m_2}}$

➤ Taking +ve value of $\tan\theta$,acute angle is found

➤ Taking $-$ve value of $\tan\theta$,obtuse angle is found

Example: Determine the acute angle between $2x + 3y - 1 = 0$ and $x - 2y + 3 = 0$.

[DU' 18-19, 08-09, JU' 18-19]

$$m_2 = -\frac{1}{-2} = \frac{1}{2}$$

$$m_1 = -\frac{2}{3}$$

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \frac{-\frac{2}{3} - \frac{1}{2}}{1 + \left(-\frac{2}{3}\right)\left(\frac{1}{2}\right)}$$

$$= -\frac{\frac{7}{6}}{2} = \boxed{-\frac{7}{4}} \to \text{obtuse}$$

$$= \frac{2}{3}$$

$$\theta = \tan^{-1}\left(-\frac{7}{4}\right)$$

$$\text{Acute Angle} = 180° - \tan^{-1}\left(\frac{7}{4}\right)$$

Ans!

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Three lines $a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$; $a_3x + b_3y + c_3 = 0$ are said to be concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$
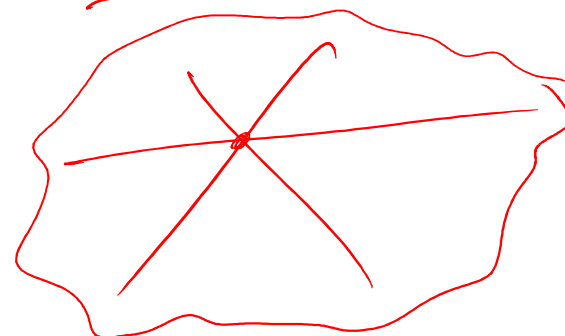
Example: If three lines $2x + by + 4 = 0$, $4x - y - 2 = 0$ and $3x + y - 1 = 0$ are concurrent then determine the value of b.   **[DU' 14-15]**

$$\begin{vmatrix} 2 & b & 4 \\ 4 & -1 & -2 \\ 3 & +1 & -1 \end{vmatrix} = 0$$

$$b = ?? \text{ H.W.}$$

Type-14: Perpendicular distance from a point

Perpendicular distance of $ax + by + c = 0$ line from the point (x_1, y_1), $d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$ units
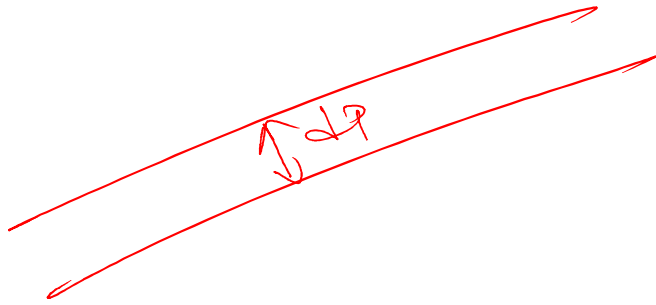
Example: Determine the length of Perpendicular on the line $4x - 3y + 1 = 0$ from the point $(-2, 1)$. **[JU' 10-11]**

$$d_p = \frac{|4(-2) - 3(1) + 1|}{\sqrt{4^2 + (-3)^2}}$$

$$= \frac{10}{5} = 2 \text{ units}.$$

Type-15: Distance between two parallel lines

Distance between two parallel line $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $d = \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$ unit

Example: Determine the distance between two parallel lines $5x + 12y + 3 = 0$ and $5x + 12y + 29 = 0$.

$$d_p = \frac{|3 - 29|}{\sqrt{5^2 + 12^2}} \text{ units}$$

$$= \frac{26}{13} \text{ units}$$

$$= 2 \text{ units Ans.}$$

Type-16: Equation of angular bisector

Equation of the bisectors of the angle formed by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

Is, $\dfrac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm \dfrac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$

$dP_1 = dP_2$

$\Rightarrow \dfrac{|a_1x+b_1y+c_1|}{\sqrt{a_1^2+b_1^2}} = \dfrac{|a_2x+b_2y+c_2|}{\sqrt{a_2^2+b_2^2}}$

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

dP_1 $P(x,y)$ dP_2

i) $a_1a_2 + b_1b_2$ → using it's sign, we will get the bisector of obtuse angle.

→ using it's opposite sign, we will get the bisector of acute angle.

ii) $a_1a_2 + b_1b_2 = 0$ → Perpendicular to each-other.

iii) c_1c_2 → using it's sign; we will get the bisector of the angle which contains the origin.

acute/obtuse

Example: Find The angular bisectors of the lines $y = 2x + 1$ and $2y - x = 4$. Also find the bisector of the angle which contains the origin ?

$$2x - y + 1 = 0 \qquad \textcircled{1} x - 2y + 4 = 0$$

i) **Acute:**

$$\frac{2x - y + 1}{\sqrt{2^2 + (-1)^2}} = \boxed{-} \; \frac{x - 2y + 4}{\sqrt{1^2 + (-2)^2}}$$

ii) **Obtuse:**

$$\frac{2x - y + 1}{\sqrt{2^2 + (-1)^2}} = \boxed{+} \; \frac{x - 2y + 4}{\sqrt{1^2 + (-2)^2}}$$
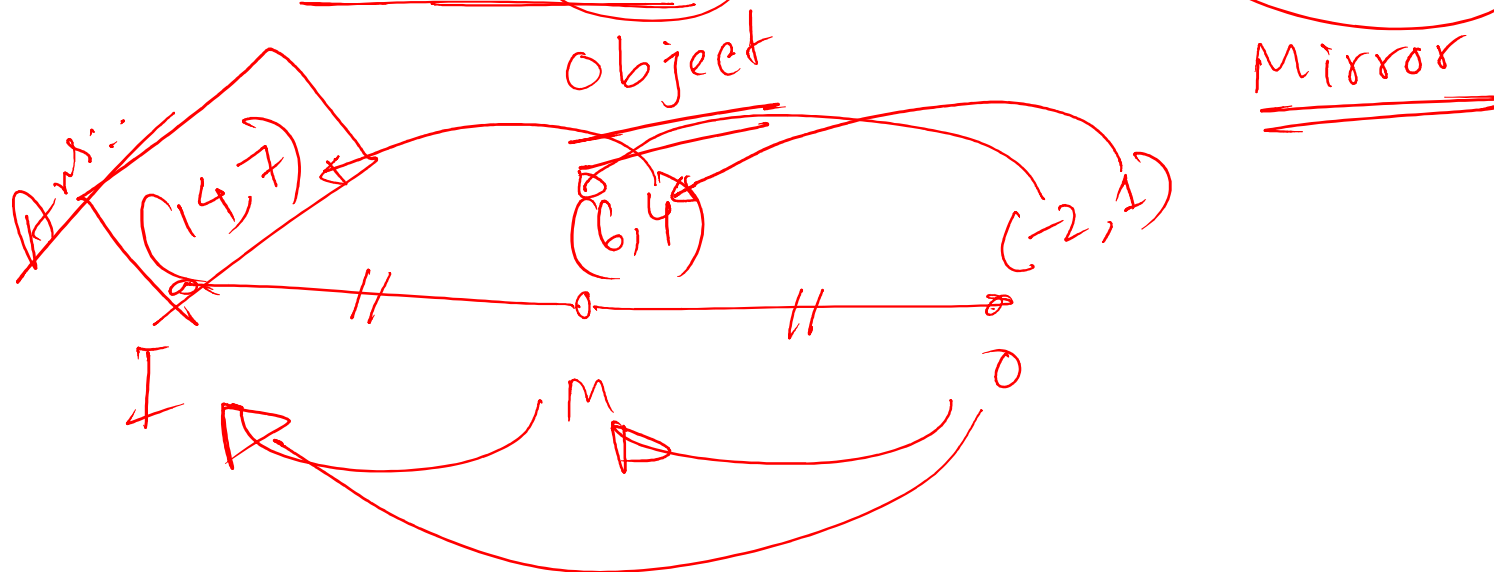
$$a_1 a_2 + b_1 b_2 = 2 \times 1 + (-1)(-2)$$
$$= +4 \longrightarrow + ve$$

is the bisector of the angle which contains the origin.

$$c_1 c_2 = (+1)(+4)$$
$$= +4$$

Type-17: Problems related to determining image

Case 1: Image of a point with respect to another point

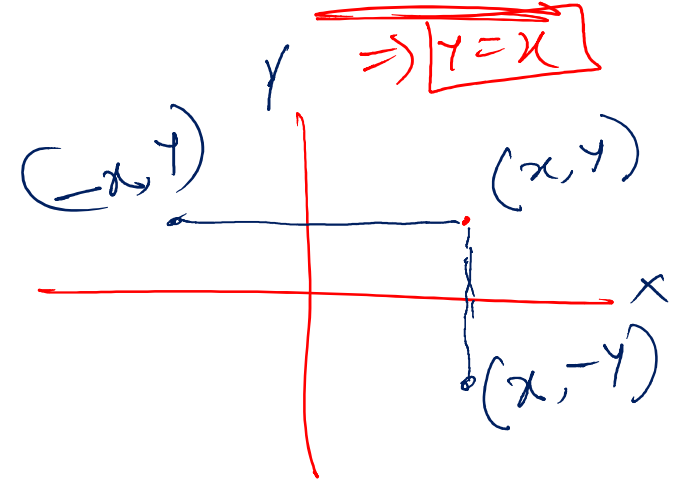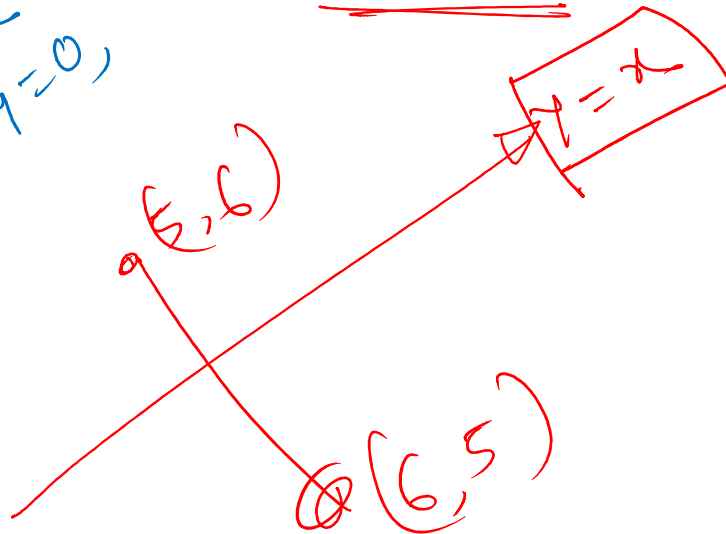**Example:** Find the image of the point (-2,1) with respect to the point (6,4) .

Type-17: Problems related to determining image

Case 2: Image of a point with respect to a straight line $\longleftarrow$ $\begin{cases} x-\text{axis} \\ y-\text{axis} \\ y=x \longrightarrow \text{line} \end{cases}$

Example: Find the image of the point (5,6) with respect to the straight line $x - y = 0$

Image of (x_1, y_1) with respect to the line $y = x$ or $x - y = 0$, is (y_1, x_1)

$(5,6)$

$y = x$

$(6,5)$

$\Rightarrow \boxed{y = x}$

$(-x, y)$

(x, y)

$(x, -y)$

Case 3: Image with respect to x or y axis

Example: Find the image of (3, −2)

    (a) With respect to the x axis $\longrightarrow$ $(3, +2)$

    (b) With respect to the y axis $\longrightarrow$ $(-3, -2)$

না বুঝে
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অভ্যাস প্রতিভাকে
ধ্বংস করে