লাল-সবুজে দাগানো

TEXT BOOK

Physics

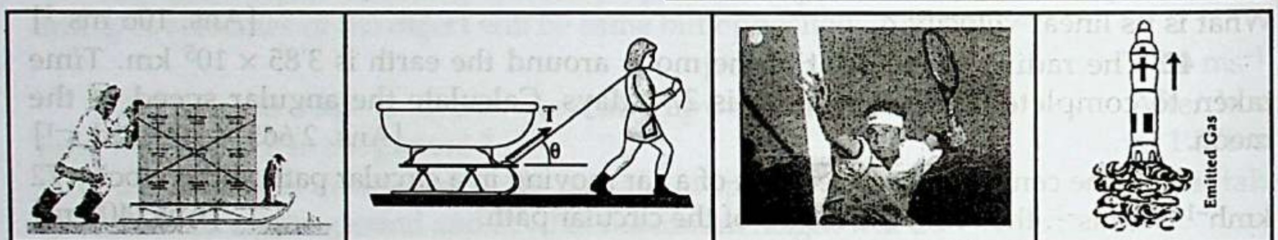
1st Paper

UNMESH

Medical & Dental Admission Care

4
NEWTONIAN MECHANICS

Key Words : Force, Fundamental forces, Momentum, Newton's Second Law of motion, Relation between Newton's law of motion, Gravitational field, Conservation of linear momentum, Moment of inertia, Angular momentum, Radius of gyration, Torque, Conservation of angular momentum, Centripetal and centrifugal force, Collision.



Introduction

Scientist Sir Isaac Newton was the first who discussed about laws of motion of bodies. Three laws discovered by Newton are the milestone of mechanics. Many problems in Physics and Engineering have been successfully solved by applying these laws. In explaining linear motion and rotational motion, momentum, conservation principle etc are some of the successes of Newtonian mechanics.

After studying this chapter students will be able to—

- explain intuitive concept of force.
- explain laws used in Newtonian mechanics by applying calculus.
- explain limitation of Newton's laws.
- explain terms relating linear and angular momentum.
- learn about the use of centripetal and centrifugal force.
- explain elastic and nonelastic collisions and will be able to solve problem relating this.

Practical : Determination of moment of inertia of a Fly Wheel.

4·1 Intuitive concept of force

We have learnt about static inertia, dynamic inertia, displacement, velocity, acceleration in the previous chapter. If you kick a football it normally moves ahead. But

Fig. 4·1 (a)

Fig. 4·1 (b)

by hitting an iron-ball of the same size, it is not possible to make similar motion. A jet plane cannot be moved alone by pushing [Fig. 4·1(a)]; but a cycle moving in a particular speed can be stopped by pulling it from behind [Fig. 4·1(b)]. From these examples it is understood that the body having large mass cannot change its inertia of rest or inertia of motion easily. So, the larger the mass, higher the inertia.

We have learnt from the above examples that in order to change the state of a body at rest or a body in motion something from outside is to be applied tangentially. In our daily life, we often push bodies by our sides, sometimes we take things from one place to another by pulling or lifting. In all cases these need to have physical contact between the object and the person who applies force. This type of force is called **contact or tangential force**. Examples of contact or tangential force are frictional force, force created due to collision, stretched force etc. It can be said that **whatever is used to change the state of rest or state of motion of a body is called force.**

Similarly, many events are happening in nature whereat two bodies are attracting or repelling each other. Again, without being nearer or in touch two bodies may be attracted by each other. For example, we do not know easily whether a mango of a tree is attracting another mango of the tree or that mango is being attracted by the earth or not. When the mango falls to the ground, it is seen that it touches the ground due to the attraction by the earth or due to its weight. This type of force of attraction is called **gravitational force.**

Again, when a box is pulled over the floor, then there is a force acting between the floor and the box which resists the motion of the box. This retarding force is called the **frictional force.** Ratio of this frictional force and reaction force is frictional co-efficient (μ). $\therefore\ \mu = \frac{f}{R}$.

In case of dynamic friction it is f_k and for static friction it is f_s and reaction R = weight of the body = mg, in case of inclined surface R = $mg \cos\theta$.

Nucleons exist closely and side by side inside the nucleus of the atom. In this case, there is a force of attraction for which nucleons do not get separated. This type of attraction in nucleus is called **nuclear force.**

In practice, there is no body on which force from outside does not act. But, if the resultant of two more forces acting on a body from outside is zero, then no effect of those forces is observed on the body. For example, if two equal forces act on a table from two sides along the same line in order to move it, the table will not move as the applied forces are equal and opposite. That means, the table will remain static. Again, if a container is placed on a table its weight (W) will act downward and the reaction force R of the table will act upward. Here W = R, hence the container will remain static [Fig. 4·2]. **All these forces are contact forces.**

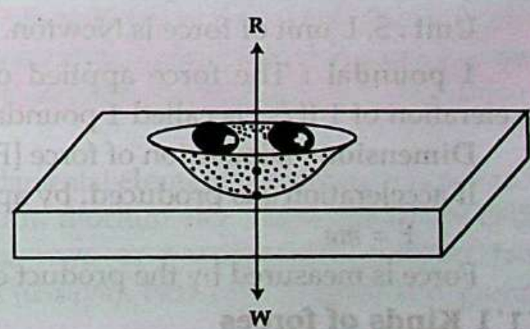


Fig. 4·2

From Newton's first law of motion we can have idea about the force. That means, if something is not applied on a body the body would like to stay in its state of rest indefinitely and also a moving body would like to move indefinitely in a straight line in perpetual motion. This property of the body is the **inertia.** So we take this decision that influence from outside for which state of a body either at rest or in motion is changed is called the force.

The body which has heavy mass will have higher inertia. If the inertia is more, large force is needed to move it.

Definition : The external reason that changes the static body into motion or changes the motion of a body or tends to change the state of rest or motion is called force.

Work :

- Strongly push a CNG baby-taxi which is at rest on a bitumin used road. What will you see ? The baby-taxi moves a little ahead.
- Now push a truck standing on the road like before. It will not move at all. Now push the truck by some people together. You will see that the truck moves ahead. What is the reason behind the difference of motion in the two cases ?

Mass of truck is many times more than that of the baby-taxi. Hence its inertia is also more. So more force is needed to move the truck than the baby-taxi.

A list of effects that the force creates on motion has been prepared. These are :

(1) Applied force can make a body at rest to move. That means force can create acceleration.

(2) Due to application of force velocity of the moving body can either be decreased or increased.

(3) Applied force can change velocity i.e., the direction of motion of the body.

(4) Forces act in couple.

Unit : S. I. unit of force is Newton. F.P.S. unit of force is poundal.

1 poundal : The force applied on a body of mass of 1 pound that generates acceleration of 1 ft/s^2 is called 1 poundal.

Dimension : Dimension of force $[F] = [MLT^{-2}]$

If acceleration a is produced, by applying force F on a body of mass m, then

$$F = ma \quad \ldots \quad \ldots \quad \ldots \quad (4.1)$$

Force is measured by the product of mass and acceleration.

4·1·1 Kinds of forces

Although we are familiar with different kinds of forces in nature and these forces have different names but all forces are not fundamental. **The forces which are fundamental or natural i.e., the forces which are not derived from other forces, rather other forces are derivatives of these forces are called fundamental forces.**

There are four fundamental forces in nature. Other forces can be explained by any one or more than one of the fundamental forces. These fundamental forces are :

1. Gravitational force
2. Electromagnetic force
3. Strong nuclear force
4. Weak nuclear force

1. Gravitational Force : In the universe, there exists a force of attraction between any two particles or objects. This attractive force is called **gravitational force.** The magnitude of this force is proportional to the product of mass of the two objects or particles and inversely proportional to the square of distance between them. Scientists believe that mutual exchange of a kind of particles between two bodies is responsible for gravitational attraction between the bodies. Particle of this kind is named **graviton**.

2. Electromagnetic Force : A kind of force exists between two charged bodies and between two magnetic materials. These are called Coulomb's electric force and magnetic force respectively. Electric force and magnetic force can either be attractive or repulsive. The electric and magnetic forces are closely related to each other. Actually, the force acting between charged particles in motion is **electromagnetic force.** When the charges are in motion then they produce magnetic field. Again, varying magnetic field acts as a source of electric field. It is assumed that due to mutual change of massless, chargeless particles called photons this force becomes active.

Electromagnetic force is demonstrated in elastic force, molecular structure, chemical reaction etc.

3. Strong nuclear force : Nucleus of an atom is composed of protons and neutrons. These are combinedly called **nucleon**. In the nucleus protons having similar charges stay very close to each other, so Coulomb's strong repulsive force is active there which could break away the nucleus. But in practice most of the nuclei are stable. Gravitational force acting between nucleons is so weak that it cannot balance the Coulomb's force. So, there must be a strong force active that keeps the nucleus as it is. This force is called **strong nuclear force**. Scientists believe that there is one type of particles called **meson** inside nucleons by whose mutual exchange this force becomes active. This force is attractive and is not active outside the nucleus; i.e., this force is active in **short range**.

4. Weak nuclear force : There are some fundamental **elements** in nature whose nuclei disintegrate spontaneously (for example, uranium, thorium etc). These nuclei are called **radioactive nuclei.** Three different types of rays or particles are emitted from radioactive nucleus. These are called alpha rays (α-rays), beta rays (β-rays) and gamma rays (γ-rays).

When beta particles or rays are emitted from radioactive nucleus, then energy is also emitted along with it. But from experimental result it is observed that the amount of energy released from the nucleus is more than the kinetic energy of beta particles. Naturally, question arises, if β-particle carries a little or small portion of energy then where the rest of the energy remains ? In 1930 **W. Pauli** proposed that rest of the energy is carried by another type of particles which are emitted along with β-particles. These particles are called **neutrino.** The emission of β-particles and neutrino particles is due to a fundamental force called **weak nuclear force.** This force is much weaker than strong nuclear force or electromagnetic force. Disintegration of many nuclei is caused due to this force. It is assumed that due to mutual exchange of particles called bosons the force becomes active.

4·1·2 Comparison of intensities of the fundamental forces

Comparison of relative strength of the four fundamental forces shows that strongest force is the strong nuclear force and the weakest one is the gravitational force.

Both strong and weak nuclear forces are **short range** forces. These forces are not active outside the nucleus. On the other hand range of gravitational and electro-magnetic force is almost infinite.

In order to have an idea about the strength of the four fundamental forces, if we take the value of strong nuclear force as 1 then relative strengths of the weak nuclear force, electromagnetic force and gravitational force are 10^{-12}, 10^{-2} and 10^{-39} respectively.

Scientists have been trying for many years to establish relation between the four fundamental forces. Prof. Abdus Salam, Wienberg and Glashow— these three scientists, after many years of research, have established a relation between weak nuclear force and electro-magnetic force which is known as **Salam-Wienberg's Theory.**

4·1·3 Momentum

Suppose for any reason there is a collision between two bodies. After collision how will you ascertain direction towards which the bodies will move ? Whether by their momentum or by their velocities ? Why the impulse of a moving rickshaw is more than that of a moving bicycle ? Why is it difficult to stop a moving rickshaw than a moving bicycle ? The reason for these events is momentum.

Then what is momentum ? It can be said that the motion produced in a body due to the combined effect of mass and velocity is the momentum of that body. Keeping mass constant, if velocity is increased, then momentum of the body also increases. If same body moves with higher velocity, then its momentum also becomes higher. As many times the velocity of a body increases same increase of force compared to previous one is needed to stop that body. If a car moves with double velocity, then double amount of force compared to previous one will be needed to stop that car. Mass of a bullet is very small, but its velocity is very large and so momentum becomes large and the impulse of the bullet is very strong.

Definition : The property of a body that is produced in it due to the combined effect of mass and velocity of the body is called momentum. It is measured by the product of mass and velocity of the body. Inertia of motion is proportional to momentum. It is a vector quantity.

Unit : S. I. unit of momentum is kg ms^{-1}

Dimension : $[P] = [MLT^{-1}]$

4·2 Newton's laws of motion

In 1687, Sir Isaac Newton in his famous and immortal book, "Natural philosophia principia Mathematica" published three laws connecting relation between mass of a body, motion and force. These three laws are known as Newton's laws of motion.

First law : **If the state of an object is not changed by applying external force, a static body will remain stationary for ever and a moving body will continue to move with uniform velocity i.e., it will be moving in straight line with uniform speed.**

Second law : **Rate of change of momentum of a body is proportional to the force applied on it and the direction in which the force is active, the momentum will also be active in that direction.**

Third law : **For every action there is an equal and opposite reaction.**

Inquisitive work : Can a passenger inside a motor car move the car by pushing it from inside ? Explain.

If a passenger sitting inside a motor car applies force on the car, then according to the third law of Newton the car will also apply equal and opposite force on the passenger. Due to the action of these two action-reaction force momentum of the system formed by the car and the passenger does not change. As a result the car does not move. So the car remains stationary.

4·2·1 Newton's second law of motion

Law : **Rate of change of momentum of a body is proportional to the force applied on it. The direction in which this force acts, the change of momentum also occurs in that direction.**

With the help of this law direction, magnitude, characteristics of force, relation between acceleration and force, unit force, unit of force and independent principle of force can be known.

Derivation of $\vec{F} = m\vec{a}$ (calculus method)

Suppose a body of mass is m and it is moving with uniform velocity $\vec{v_0}$ [Fig. 4·3].

Suppose a constant force $\vec{F}$ is acting for time t on the body along its direction of motion. Consequently velocity of the body changes to $\vec{v}$.

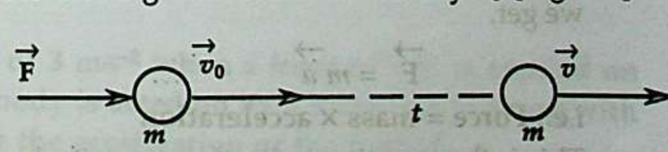


Fig. 4·3

So, momentum of the body moving with velocity $\vec{v}$ is

$$\vec{P} = m\vec{v} \quad \ldots \quad \ldots \quad \ldots (4.2)$$

Now, rate of change of momentum $\dfrac{d\vec{P}}{dt} = \dfrac{d}{dt}(m\vec{v})$

Rate of change of momentum is proportional to applied force.

$$\therefore \quad \vec{F} \propto \frac{d\vec{P}}{dt}$$

$$= k\frac{d\vec{P}}{dt} = k\frac{d}{dt}(m\vec{v})$$

$$\therefore \quad \vec{F} = km\frac{d\vec{v}}{dt} = km\vec{a} \quad \ldots \quad \ldots \quad (4.3) \quad \left[\text{Here } k = \text{constant}, \frac{d\vec{v}}{dt} = \vec{a}\right]$$

From the definition of unit force it can be shown that $k = 1$

When $m = 1$ unit, $|\vec{a}| = 1$ unit, then $|\vec{F}| = 1$ unit.

$\therefore$ by inserting these values in equation (4.3) we get,

$$1 = k \,.\, 1 \times 1$$

$$\therefore \quad k = 1$$

$$\therefore \quad \vec{F} = m\vec{a} \quad \ldots \quad \ldots \quad \ldots \quad (4.4)$$

Instead of a single force if forces $\vec{F}_1, \vec{F}_2, \vec{F}_3 \ldots \vec{F}_n$ etc are applied on the body then net force active on the body $= \Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \ldots + \vec{F}_n$.

$$\therefore \quad \text{Newton's second law is } \Sigma\vec{F} = m\vec{a} \quad \ldots \quad \ldots \quad [4.4(a)]$$

Here direction of acceleration is along the net force. We can define unit force from Newton's second law.

The force applied on a body of unit mass to produce unit acceleration in it is called unit force.

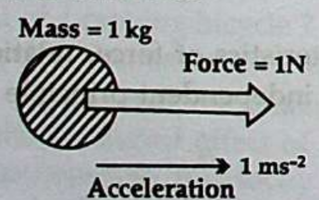


Fig. 4·4

That means,

$m = 1$ kg (S. I. Method)

$|a| = 1$ ms^{-2} then

F = 1 N, [Fig. 4·4]

So, by inserting these values in equation (4.3),

we get,

$$\vec{F} = m\vec{a} \quad \ldots \quad \ldots \quad \ldots \quad (4.5)$$

i.e., **Force = mass × acceleration**

This is the equation indicating magnitude of force.

4·2·2 Independent principle of forces

According to Newton's second law of motion rate of change of momentum a body will be active along the direction of force. So, the momentum that exists along the direction of force will be changed with time only. Even in case of many forces effect of one force on the body will not be influenced by other forces. The characteristic or feature of the forces on a body is called the **independent principle of forces** or **physical non-dependence.**

What we have learnt from Newton's second law of motion are :

(i) Acceleration of a body is proportional to the applied force on the body.

(ii) When the force does not act the body does not have acceleration or retardation.

(iii) Direction of force is the direction of acceleration.

(iv) When force acts on a body it moves with acceleration.

Mathematical examples

1. A body was at rest. A force of 15 N acted on it for 4 sec and then that force did not act. The body then moved 54 m in 9 sec. Find the mass of the body.

Here,
$F = 15$ N
$t_1 = 4$ s
$t_2 = 9$ s
$s = 54$ m
$v_0 = 0$ (since the body is at rest)
$m = ?$

Since the force did not act on the body after 4 sec, so it moved next 9 s at uniform velocity.

$$\therefore\ v = \frac{s}{t_2} = \frac{54}{9} = 6\ \text{ms}^{-1}$$

We know, $v = v_0 + at_1$

or, $6 = 0 + a \times 4$

or, $6 = 4a$

or, $a = \frac{6}{4}$

$\therefore\ a = 1{\cdot}5\ \text{ms}^{-2}$

Again, $F = ma$

or, $15 = m \times 1{\cdot}5$

or, $m = \frac{15}{1{\cdot}5}$

$\therefore\ m = 10$ kg

2. A body acquires an acceleration of 3 ms⁻² when a force of 7 N is applied on it. What is the mass of the body ? If the body is acted on by a force of 5 N along with 7 N force at an angle of 60°, what will be the acceleration of the body ?

[Ch. B. 2010; R. B. 2009; S. B. 2003]

1st part

Here,
$F = 7$ N
$a = 3\ \text{ms}^{-2}$
$m = ?$

We know,

$F = ma$

or, $7 = m \times 3$

$\therefore\ m = \frac{7}{3} = 2{\cdot}33$ kg

2nd part

Let the resultant force be R.

Here,
$P = 7\text{ N}$
$Q = 5\text{ N}$
$\alpha = 60°$

Now, $R = (P^2 + Q^2 + 2PQ\cos\alpha)^{\frac{1}{2}}$

$\therefore\ R = (7^2 + 5^2 + 2 \times 7 \times 5 \times \cos 60°)^{\frac{1}{2}}$

$= (49 + 25 + 2 \times 7 \times 5 \times \frac{1}{2})^{\frac{1}{2}}$

$= (74 + 35)^{\frac{1}{2}} = (109)^{\frac{1}{2}}$

$= 10{\cdot}44\text{ N}$

Again,

Here,
$R = 10{\cdot}44\text{ N}$
$m = 2{\cdot}33\text{ kg}$
$a' = ?$

$R = ma'$

$\therefore\ a' = \frac{R}{m} = \frac{10{\cdot}44}{2{\cdot}33}\text{ ms}^{-2}$

$= 4{\cdot}48\text{ ms}^{-2}$

Ans. Mass of the body is $2{\cdot}33$ kg and acceleration is $4{\cdot}49\text{ ms}^{-2}$.

3. A force of 10 N acts on a stationary body of mass of 2 kg. If the action of the force stops after 4 s, then how far will the body travel from the start in 8 s ?

Here,
force, $F = 10\text{ N}$
mass, $m = 2\text{ kg}$
In first case,
initial velocity, $v_0 = 0$
time, $t = 4\text{ s}$
acceleration, $a = ?$
distance, $s_1 = ?$

We know,

$F = ma$

or, $a = \frac{F}{m} = \frac{10}{2} = 5\text{ ms}^{-2}$

$\therefore$ distance travelled in first 4 s,

$s_1 = v_0 t + \frac{1}{2}at^2$

$= 0 \times 4 + \frac{1}{2} \times 5 \times 4^2$

$= 40\text{ m}$

In second case,
velocity, $v = 20\text{ ms}^{-1}$
time, $t = 4\text{ s}$
distance, $s_2 = ?$

the velocity of the body after first 4 s,

$v = v_0 + at = 0 + 5 \times 4 = 20\text{ ms}^{-1}$

distance travelled in next 4 s,

$s_2 = vt = 20 \times 4 = 80\text{ m}$

$\therefore$ total distance travelled in 8 s from the start,

$s = s_1 + s_2 = 40 + 80 = 120\text{ m}$

4. How much force is to be applied on a body of mass of 60 kg so that its velocity increases to 10 ms^{-1} in 1 minute ?

We know,

$$F = ma = \frac{m\Delta v}{t}$$

$$= \frac{60 \times 10}{60} = 10 \text{ N}$$

Here,

mass, $m = 60$ kg

time, $t = 1$ min. $= 60$ sec

increase of velocity,

$\Delta v = 10$ ms^{-1}

5. How much force is to be applied on a body of weight of 980 N to give acceleration of 1 ms^{-2} ?

We know,

$$W = mg$$

$$\text{or, } m = \frac{W}{g} = \frac{980}{9{\cdot}8} = 100 \text{ kg}$$

Here,

weight of the body,

W = 980 N

acceleration, $a = 1$ ms^{-2}

F = ?

Again,

$$F = ma = 100 \times 1 = 100 \text{ N}$$

6. A bullet of mass of 14 g of a rifle at the velocity of 3·6 ms^{-1} can penetrate 0·21 m thick block of wood.

We know,

$$\text{work done, } W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$\text{or, } Fs = \frac{1}{2}m(v^2 - v_0^2)$$

$$\text{or, } F = \frac{\frac{1}{2} \times 14 \times 10^{-3}\,[0 - (3{\cdot}6)^2]}{0{\cdot}21}$$

$$= -\,0{\cdot}432 \text{ N}$$

Here,

$m = 14$ g $= 14 \times 10^{-3}$ kg

$v_0 = 3{\cdot}6$ ms^{-1}

$v = 0$

$s = 0{\cdot}21$ m

7. How much force is to be applied on a body of mass of 4 kg to move it with acceleration of 10 ms^{-2} ? (Frictional force of the road is 2·5 N kg^{-1}] [B. B. 2001]

We know,

effective force, $F = P - F_k$

$\therefore$ $40 = P - 10$

or, $P = 50$ N

$\therefore$ applied force = 50N

Here,

mass of the body, $m = 4$ kg

acceleration, $a = 10$ ms^{-2}

$\therefore$ effective force, $F = ma = 4 \times 10 = 40$N

Frictional force = 2·5 N kg^{-1}

$\therefore$ total frictional force, $F_K = 2{\cdot}5 \times 4 = 10$N

applied force, P = ?

8. A box of mass of 70 kg is being pulled on the floor with a horizontal force of 500 N. When the box moves the frictional coefficient between the box and the floor is 0·50. Calculate the acceleration of the box. [D. B. 2011; S. B. 2009; Din. B. 2009; R. B. 2007; J. B. 2004]

Here,

$m = 70$ kg

$\mu_k = 0{\cdot}50$

normal reaction,

$R = 70 \times 9{\cdot}8$ N $= 686$ N

horizontal force, $F_1 = 500$ N

acceleration, $a = ?$

We know,

Frictional force, $F_k = \mu_k R$

or, $F_k = 0{\cdot}50 \times 686$ N

$= 343$ N

Again,

resultant force,

$F = F_1 - F_k$

$= (500 - 343)$ N

$= 157$ N

Now, $F = ma$

or, $a = \frac{F}{m} = \frac{157 \text{ N}}{70 \text{ kg}}$

$= 2{\cdot}24 \text{ ms}^{-2}$

Work : Why a player in a cricket match pulls his hand backward to catch the ball.

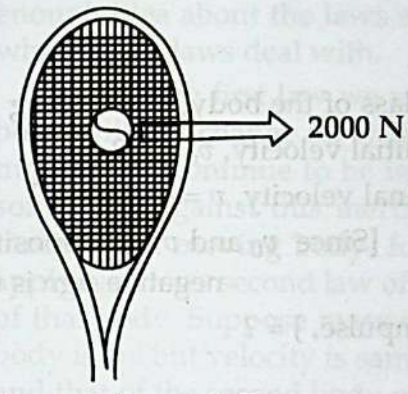
According to Newton's second law if the applied force is less, then acceleration will be less. If the change of velocity is less, time taken for this change is more, acceleration will be less accordingly. So, in cricket match a player pulls backward his hand to catch the ball so that for a particular change of velocity time taken is more; consequently acceleration and reaction force will be less.

4·2·3 Impulsive force

During collision, explosion, sudden strike etc such type of force acts. To strike carom globule by a striker, hitting of a tennis ball by the bat, to kick a football, to hit a nail by a hammer, striking of a string of a musical instrument etc are special types of force. It is called **impulsive force.**

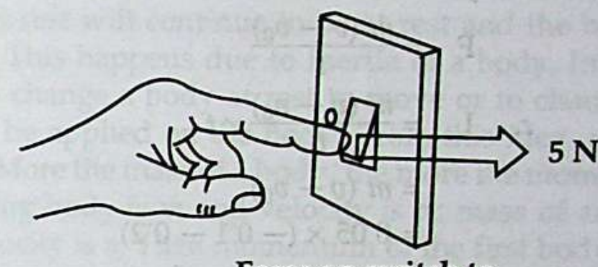
Impulsive force acts for such a brief period that during that time displacement of the body may be ignored. But as the magnitude of force is very large, so there is a sudden change of velocity and along with it momentum also changes. It is not possible to know or measure impulsive force correctly. It is not needed also. If change of momentum can be measured i.e., if impulse of force is known, then total result of the force will be known. For this reason, **this sype of force is called impulsive force.** That means, **for a very brief period if a large force acts on a body, then that force is called impulsive force.**

Example : Suppose, when a tennis ball is struck by a racket then a strong force acts on the tennis ball. Here the time of collision between the ball and the racket is



Force on tennis ball

Fig. 4·5 (a)



Force on switch to put on light

Fig. 4·5 (b)

very small. This type of force is impulsive force [Fig. 4·5(a)]. Again, when electric switch is made off and on, this impulsive force also becomes effective [Fig. 4·5(b)].

4·2·4 Impulse of force

Product of a force and the duration of action of the force is called impulse of that force. If force $\vec{F}$ is acting on a body for time t, then

impulse of force, $\vec{J} = \vec{F} \times t$ ... ... (4.6)

$$= \frac{m\vec{v} - m\vec{v_0}}{t} \times t = m\vec{v} - m\vec{v_0} = \text{change of momentum}$$

∴ Impulse of force is equal to the change of momentum.

Mathematical examples

1. A force of 16 N acts on a body of mass of 4 kg for 4 s. Determine (a) the change of velocity and (b) the impulse of force.

Here,

Force, $\vec{F} = 16$ N

$m = 4$ kg

$t = 4$ s

(a) Suppose, change of velocity $= \vec{v} - \vec{v_0}$

We know,

Impulse of force = change of momentum

$\vec{F} \times t = m\vec{v} - m\vec{v_0}$

or, $F \times t = m(v - v_0)$

$\therefore (v - v_0) = \frac{F \times t}{m} = \frac{16\text{N} \times 4\text{ s}}{4\text{ kg}} = 16 \text{ ms}^{-1}$

(b) impulse of force, $J = F \times t = 16\text{N} \times 4\text{ s}$

$= 64$ Ns

2. A body of mass 0·05 kg hits a vertical wall with a horizontal velocity of 0·2 ms^{-1} and rebounds with a velocity of 0·1 ms^{-1}. Find the impulse.

Suppose, impulse of force = J

We get,

$$J = F \times t \text{ and}$$

$$F = \frac{m(v - v_0)}{t}$$

$$\therefore J = \frac{m(v - v_0)}{t} \times t$$

$$= m(v - v_0)$$

$$= 0{\cdot}05 \times (-0{\cdot}1 - 0{\cdot}2)$$

$$\therefore J = -0{\cdot}015 \text{ kg-ms}^{-1}$$

[$-$ve sign indicates that J and v are in the same direction]

$$\therefore |J| = 0{\cdot}015 \text{ kg-ms}^{-1}$$

Here,

Mass of the body, $m = 0{\cdot}05$ kg

Initial velocity, $v_0 = 0{\cdot}2$ ms^{-1}

Final velocity, $v = -0{\cdot}1$ ms^{-1}

[Since v_0 and v are opposite, so negative sign is used]

Impulse, J = ?

3. A cyclist while cycling at velocity 8 ms^{-1} stopped pedalling and observed that after travelling a distance 48 m the cycle stopped. Calculate the frictional force between the tyre of the cycle and the road and impulse in 2 secs. [mass of the cyclist including the cycle = 147 kg]

Let the frictional force = F and

retardation produced due to force, F = a

We get, $v^2 = v_0^2 - 2as$ ... ... (i)

$\therefore$ from equation (i) we get,

$$F = ma = \frac{m(v_0^2 - v^2)}{2s}$$

$$= 147 \text{ kg} \times \frac{\{(8 \text{ ms}^{-1})^2 - 0\}}{2 \times 49 \text{ m}} = 96 \text{ N}$$

Here,

$v = 0$

$v_0 = 8$ ms^{-1}

$m = 147$ kg

$s = 49$ m

$\therefore$ frictional force, F = 96 N and impulse of force in 2 sec = F × t = 96 × 2 = 192 Ns

4. A cricket ball of mass of 150 g coming at velocity 10 ms^{-1} is struck by a bat. The ball returned at velocity 18 ms^{-1}. If the duration of collision between bat and ball is 0·1 s, then find the magnitude of the applied force on the cricket ball.

We know,

$$F \times t = mv - mu$$

$$F = \frac{m(v - v_0)}{t}$$

$$= \frac{0{\cdot}15 \times (-18 - 10)}{{\cdot}01}$$

$$= -420 \text{ N}$$

$$\therefore |F| = 420 \text{ N}$$

Here,

initial velocity, $v_0 = 10$ ms^{-1}

mass of the cricket ball,

$m = 150$ g $= 0{\cdot}15$ kg

final velocity, $v = -18$ ms^{-1}

applied force, F = ?

4·3 Relation between Newton's laws of motion

In order to establish relation between Newton's laws of motion we should have enough idea about the laws and also we should have knowledge about the subjects on which those laws deal with.

From the first law we understand that if no external influence acts on a body the body will not change its state. Body at rest will continue to be at rest and the body in motion will continue to be in motion. This happens due to inertia of a body. In doing something against this inertia, i.e., to change a body at rest to move or to change the motion of a moving body, force is to be applied on the body. From this idea, we can apply Newton's second law of motion. More the mass of a body, the more the momentum of that body. Suppose mass of a moving body is m and velocity is v; mass of another body is $2m$ but velocity is same i.e., velocity is v. Then momentum of the first body = mv and that of the second body = $2\ mv$. By resisting i.e., by applying force if the two bodies are stopped at the same time then change of momentum of the second body will be double than that of the first body. From the second law, we know that rate of change of momentum is proportional to the applied force. So, in order to stop the second body at the same time double amount of force is to be applied. Again, if two equal forces (F) are applied on the two bodies then acceleration of the first body is a_1 and acceleration of the second body is a_2, then according to Newton's second law, $F = ma_1$ and $F = 2\ ma_2$.

So, it is seen that there is a relation between momentum and acceleration and by this a relation can be established between the first and the second law of Newton or one law can be transformed into another law.

In other way, if forces F_1 and F_2 are applied on two bodies along the same straight line then while moving it may happen that the two bodies might have collided with each other. Whenever they are collided then second body applies equal and opposite reaction force on the first body. In this case, the force by which the second body is acted is called **action force** and this body after collision applies force on the first body in the opposite direction which is called **reaction force**. From Newton's third law of motion it is known that this action and reaction are equal.

From the above event it is observed that inertia of the bodies, creation of acceleration due to the application of force, action and reaction all these activities are mutually related with Newton's first, second and third laws.

Mutual relation among Newton's laws of motion can be established mathematically in the following way

■ **Relation between second law and first law**

We know from Newton's second law of motion that rate of change of momentum is proportional to the applied force. That means,

$$\frac{m\vec{v} - m\vec{v_0}}{t} \propto \vec{F}$$

$$\therefore \quad \frac{m(\vec{v} - \vec{v_0})}{t} \propto \vec{F}$$

or, If $m\vec{a} = k\vec{F}$, $k = 1$, then

$\vec{F} = m\vec{a}$; here $\vec{F}$ = applied force, $\vec{a}$ = acceleration, $\vec{v}_0$ = initial velocity, $\vec{v}$ = final velocity

If external force is not applied, then $\vec{F} = 0$ and $\vec{a} = 0$.

But since mass of a body is not zero, i.e., $m \neq 0$,

so, $\vec{a} = \dfrac{d\vec{v}}{dt} = 0$, i.e., $\vec{v}$ = constant ... (4.7)

So, it can be said that if no external force is applied, there is no change in velocity. No change takes place in the state at rest or in the state of motion. That means, in absence of external force momentum of particles always remains same or constant.

■ **Relation between first law and third law :**

From Newton's first law we know that if external force does not work momentum remains constant. That means

Momentum, $\vec{P} = m\vec{v}$ = constant ... ... (4.8)

By differentiating with respect to t we get,

$$\therefore \frac{d\vec{P}}{dt} = m\frac{d(\vec{v})}{dt} \qquad \ldots \quad \ldots \quad \ldots \quad (4.9)$$

Again, when one of the two bodies applies force on the other body, then rate of change of resultant momentum becomes equal and opposite.

$$\therefore \frac{d\vec{P}_1}{dt} = -\frac{d\vec{P}_2}{dt}$$

$$\frac{d}{dt}(m_1\vec{v}_1) = -\frac{d}{dt}(m_2\vec{v}_2) \qquad \ldots \quad \ldots \quad \ldots \quad [4.9(a)]$$

or, $m_1\vec{a}_1 = m_2\vec{a}_2$ or, $\vec{F}_1 = -\vec{F}_2$, i.e. Action force = Reaction force

By equation [4.9(a)] mutual relations between Newton's First law and Thrid law of motion can be established.

■ **Relation between second law and third law :**

We know from Newton's second law of motion that the rate of change of momentum is the applied force. If impulsive force is considered then it can be written

Impulsive force = rate of change of momentum.

In this case, the force for which impulse is produced, conversely for that force counter impulse is produced. Here it can be said,

action = reaction.

This is Newton's third law.

4·4 Applications of Newton's laws of motion

When a body applies force on another body then the second body also applies equal and opposite force on the first body. We have learnt about this action and reaction forces from Newton's third law of motion. In nature forces act in couples. In nature there is no individual separate force. Two forces are complementary to each other. One of these forces is called **action force** and the other one is **reaction force.** As long as action force is there, reaction force also exists there. Some practical applications of Newton's laws of motion are described below with examples.

1. Motion of a carriage

When a carriage moves on the road, then force $\vec{F}$ applied in the belt on the shoulder of the horse or on the arms moves the carriage forward; at the same time carriage also pulls the horse backward with equal and opposite force $\vec{F}$. Naturally, question arises how the carriage moves forward ? Look at the picture below.

How the carriage moves forward with passengers ? : In order to move the carriage the horse applies force on the ground obliquely. Simultaneously ground also applies equal and opposite reaction force R on the horse. This force can be resolved into horizontal F_H and vertical F_V components. Vertical components F_V balances weight of the horse. Now if the horizontal component F_H becomes greater than reaction force R applied backward by the carriage, then due to the action of the force $F_H - R$ horse moves forward i.e., the carriage moves ahead [Fig. 4·6].

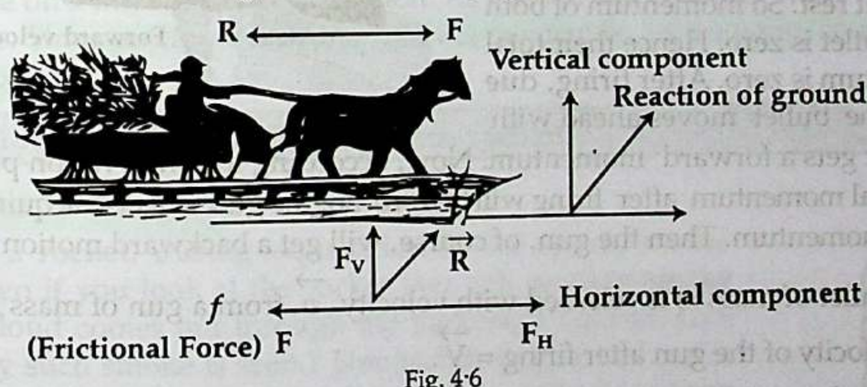


Fig. 4·6

Now if the motion of the carriage is considered separately, then we will find that two forces act on it—

(i) Frictional force f on the wheel due to its contact with the ground. This force resists the motion of the carriage.

(ii) Force applied by the horse F; this force tries to pull the carriage ahead.

2. Pulling a boat by rope (or Towing a boat)

Suppose M is a boat. At point O a rope is fastened and the boat is pulled by the rope along the bank of the river OR with a force $\vec{F}$. F can be resolved at point O into two components— horizontal and vertical components [Fig. 4·7].

Horizontal component = F cos θ, its direction is along OA and the vertical component = F sin θ, its direction is along OB.

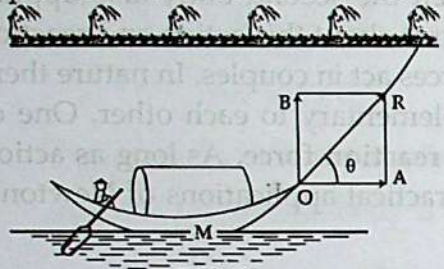


Fig. 4·7

Horizontal component of the force F cos θ pulls the boat forward and the vertical component F sin θ pulls the boat towards the bank of the river. But vertical component F sin θ is repulsed by the helm. Larger the rope of the boat, smaller the value of angle θ; consequently value of F sin θ will be less and that of F cos θ will be more. So, the boat will move fast ahead.

Work : How the motion of a boat increases in case of towing.

3. Long jump of an athlet

Before long jump an athlet runs some distance from behind. The purpose of it is to acquire inertia of motion so that he can travel some distance after jump.

4. Firing of a bullet from a gun

When a bullet is fired from a gun, the bullet moves ahead with tremendous speed. If the gun applies force F on the bullet, the bullet also applies equal and opposite force on the gun. Due to this reaction force the gun also recoils backward [Fig. 4·8].

This can be explained by momentum as well. Before firing both the gun and the bullet remain at rest. So momentum of both the gun and bullet is zero. Hence their total initial momentum is zero. After firing, due to explosion, the bullet moves ahead with a velocity. So it gets a forward momentum. Now, according to conservation principle of momentum total momentum after firing will be zero. So, the gun will also acquire an equal and opposite momentum. Then the gun, of course, will get a backward motion [Fig. 4·8].

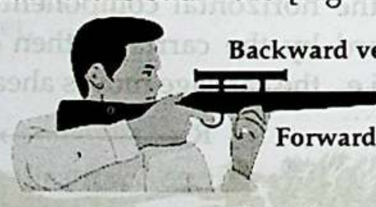


Fig. 4·8

Let the bullet of mass m is released with velocity $\vec{v}$ from a gun of mass M. Again, suppose the velocity of the gun after firing = $\vec{V}$.

∴ Before firing their total momentum = 0

After firing their total momentum

= momentum of the gun + momentum of the bullet = $M\vec{V} + m\vec{v}$

But according to the conservation principle momentum before and after must be equal.

$$\therefore \quad M\vec{V} + m\vec{v} = 0$$

$$m\vec{v} = -M\vec{V} = M(-\vec{V})$$

$$\text{or,} \quad \vec{V} = \frac{-mv}{M} \qquad \dots \qquad \dots \qquad \dots \qquad (4.10)$$

The gun is pushed back with this velocity.

According to equation (4.10) mass of the bullet × velocity of the bullet
= mass of the gun × recoil velocity of the gun.

From this equation it can be said that velocity of the bullet > recoil velocity of the gun.

5. Flying of a bird

When a bird flies along OA then the bird applies forces by its two wings over air along OB and OC. At the same time air also applies reaction force on the bird along OE and OD [Fig. 4·9].

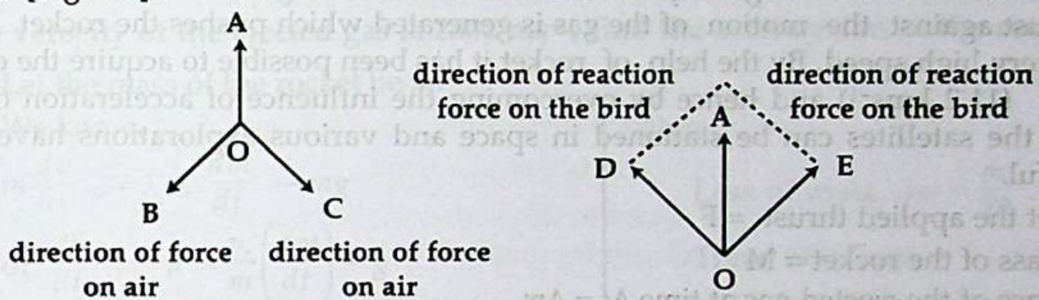


Fig. 4·9

Due to the action of these two reaction forces motion of the bird is created. The resultant of the two reaction forces is OA, so the bird can fly along OA. Now if the bird applies less force by a wing compared to the other one, then the resultant of the reaction force does not act along OA. Rather, the direction along which less force is applied by the wing, the bird inclines to that direction. As a result the direction of the motion of the bird changes. Since reaction force does not act in vacuum on the wing, so a bird cannot fly in vacuum.

Perceptional work : Why a bird cannot fly in vacuum ?

6. Exploration in the space :

When a rocket, during exploration in the space, moves upward, then if you look at the rocket you will see that smoke like white cloud comes out through the backward nozzles. Can you say why such smoke is seen ? Due to burning of fuel gas is formed at very high pressure. We often see this coil of gas from the earth. This gas comes out through the small opening at the rear side of the rocket with tremendous velocity. Due to this, a tremendous reaction force is produced which pushes the rocket forward with a very strong velocity [Fig. 4·10].

Fig. 4·10

Extensive uses of artificial satellites are found in modern telecommunication system and artificial satellites have also great contribution in space research. Behind this successive development of rocket technology is playing a vital role.

Although gas is lighter but due to very high velocity momentum of the emitted gas is very high. According to conservation principle of momentum the rocket also acquires equal but oppositely directed momentum and hence rises upward with high velocity. Normally, as fuel rocket uses liquid hydrozen and for ignition liquid oxygen is used. By a special process and controlled rate liquid hydrozen and oxygen are allowed to enter into the combustion chamber. Due to burning of the fuel high pressure gas is produced that comes out at a very high speed through the opening at the bottom of the rocket and the rocket moves ahead fast.

Let us consider that a rocket is in motion in the space. So we can ignore air resistance and influence of gravity. Since due to emission of gas from the rocket a force or a thrust against the motion of the gas is generated which pushes the rocket ahead with a very high speed. By the help of rocket it has been possible to acquire the escape velocity (11·2 kms^{-1}) and hence by overcoming the influence of acceleration due to gravity the satellites can be stationed in space and various explorations have been successful.

Let the applied thrust = F

Mass of the rocket = M

Mass of the ejected gas at time $\Delta t = \Delta m$

Escape velocity of gas = v

Change of momentum of the gas in time interval, $\Delta t = (\Delta m)v$

According to conservation principle of momentum,

change of momentum in time Δt = applied impulse force on the rocket.

$$\therefore \quad (\Delta m)v = F \times \Delta t$$

$$F = \left(\frac{\Delta m}{\Delta t}\right) v, \text{ here } \frac{\Delta m}{\Delta t} = \text{rate of using of fuel}$$

If the instantaneous acceleration of the rocket is a, then $F = Ma$

$$a = \frac{F}{M} = \frac{1}{M}\left(\frac{\Delta m}{\Delta t}\right) v \qquad \ldots \qquad \ldots \qquad \ldots \qquad (4.11)$$

with this acceleration the rocket moves forward.

From equation (4.11), it is seen that

- If mass of the rocket decreases the velocity increases.
- To increase the acceleration of the rocket, rate of ejection of gas is to be increased.
- When relative velocity of the gas increases acceleration also increases.

Mathematical examples

1. A rocket consumes 0·07 kg fuel per second. If the velocity of the gas ejected by the rocket is 100 kms^{-1}, what is the force acting on the rocket ? (Ignore the gravitational force).

Here,

Consumption of fuel per second,

$$\frac{dm}{dt} = 0{\cdot}07 \text{ kg s}^{-1}$$

Velocity of the ejected gas,

$$v_r = 100 \text{ kms}^{-1} = 1 \times 10^5 \text{ ms}^{-1}$$

We know, $F = m\frac{dv}{dt} = v_r\frac{dm}{dt} - mg$

Since there is no effect of gravitational force ($g = 0$), active force on the rocket,

$$F = v_r\frac{dm}{dt}$$

$$\therefore\ F = 1 \times 10^5 \times 0{\cdot}07 = 7 \times 10^3\ \text{N}$$

2. A rocket while moving upward loses $\frac{1}{50}$ th part of its mass in the first 2 sec. If the velocity of the ejected gas is 2500 ms^{-1}, find the acceleration of the rocket.

Let the mass of the rocket be m.

We know,

$$m\frac{dv}{dt} = v_r\frac{dm}{dt} - mg$$

$$\text{or, } \frac{dv}{dt} = a = \frac{v_r}{m}\left(\frac{dm}{dt}\right) - g$$

$$\therefore\ a = \frac{2500\ \text{ms}^{-1}}{m} \cdot \frac{m}{50 \times 2\ \text{s}} - 9{\cdot}8\ \text{ms}^{-2}$$

$$= 25\ \text{ms}^{-2} - 9{\cdot}8\ \text{ms}^{-2}$$

$$= 15{\cdot}2\ \text{ms}^{-2}$$

Here,

Loss of mass, $dm = \frac{m}{50}$

Time, $dt = 2$ sec

Velocity of the gas, $v_r = 2500$ ms^{-1}

Acceleration of the rocket, a = ?

4·5 Contribution of Newton's laws of motion

Mechanics which has been created and developed on the basis of Newton's laws of motion is called Newtonian mechanics or Classical mechanics. By the help of this mechanics motion of bodies on earth and motion of stars and planets in the vast endless sky can be analysed. By applying Newton's laws of motion we can get accurate solutions of the motion of these bodies. Newtonian mechanics or Classical mechanics has achieved remarkable success in analysing the motion of these bodies. For this reason, Newtonian mechanics is said to have laid the foundation of modern physics.

It has been assumed in Newtonian mechanics that mass and length of a body do not depend on the velocity of the body. Furthermore, it has been assumed that action or performance of measuring instruments is not influenced by their motion. In this mechanics both space and time have been considered to be constant and are not relative with respect to anything else. In Newton's first law of motion only inertia can be correctly expressed with respect to a reference. For this reason, in Newtonian mechanics a particular absolute reference of rest is to be considered.

But scientist Einstein after extensive research came to the conclusion that no absolute rest is there in the universe. All are in motion, but it may appear that a particular body is at rest with reference to another body. He further proved that laws of physics expressed in any frame of reference remain unchanged. This is the fundamental concept of the theory of relativity of Einstein. In order to make it applicable in all cases, scientist Einstein changed many Newtonian equations of motion. He proved these changes by different experiments. One of these is the fact that the mass of a body depends on speed. So, the acceleration that occurs due to the application of force does not remain constant i.e., acceleration of a body depends on its speed. Besides, length of a body and time interval also depend on motion. Experimental results of different measurement also depend on motion. Einstein's theory of relativity has generated many new concepts in physics and has changed many old ideas. Newtonian mechanics is not effective in those cases where velocity of bodies is almost close to the velocity of light. In analysing this type of high speed or motion relativistic mechanics is needed.

In analysing motions of atoms and molecules Newtonian mechanics is not effective. Quantum mechanics is needed to analyse the motion of such tiny particles. But that does not mean that classical mechanics has become obsolete.

Finally let us find the mechanism to determine the force acting on particles from the properties of the particles and their environment.

As a matter of fact, Newton's first law is related to frame of reference. Because generally the accleration of a body measured with reference to a frame of reference depends on that reference. First law states that, if there is no body available nearby, then a family of frame of reference could be obtained whereat no acceleration of a particle exists. In the absence of applied force bodies remain in the state of rest or maintain uniform linear motion— this property is nothing but inertia.

From Newton's second law of motion we have learnt that acceleration of a given body is proportional to the applied force on it. Now our question is whether same force on other bodies will be different or not. The remaining significant question is : will the same force acting on different bodies be different ? That means what kind of action is applied by the same force on different bodies. From this law we get qualitative answer of it. From Newton's third law we learn that a unit force is simply the direction of the interaction between two bodies. Further we see that the magnitude of these two forces are equal but oppositely directed. So, there is no existence of an insulated or isolated force— it is impossible to get it.

Physics is not the combination of rigid theories— rather it is a continuously developing science. It is seen that physics started attaining perfection in 1660 by Newtonian mechanics, in 1870 by Maxwell's electromagnetic theory, in 1905 by Einstein's theory of relativity and in 1925 by quantum mechanics. It is seen that theories of Newtonian mechanics are applicable in limited area and has some limitations.

During the last few decades it has been possible to measure properties of tiny speedy particles like electron, proton and other fundamental particles by quantum mechanics. Newtonian mechanics could not describe the motion of such speedy particles. Newtonian mechanics is very suitable in explaining difficult phenomena at $\frac{v}{c} \ll 1$ but collision, decay and interaction of fundamental particles having high speed cannot be explained by Newtonian mechanics. Even then importance of Newtonian mechanics is not least. Newtonian mechanics may be considered as a special case of the general mechanics that deals with particles having the speed of that of light. The mass of those materials which we deal with in our daily life is much higher than that of electron (mass of electron, $m = 9{\cdot}1 \times 10^{-31}$kg). It is a fascinating matter that the concept of extremely close relation with 'particle' is the foundation of classical mechanics.

By this mechanics or by Newton's equations of motion it is not possible to measure position x of particles and their velocity v simultaneously and accurately. There is an uncertainty. This uncertainty is known as **Heisenberg's uncertainty principle which is** expressed as :

$$\Delta x \cong \frac{h}{m \Delta v_x}, \text{ here } h = \text{Planck's constants}$$

Newtonian mechanics is a special form of general theory which fails to explain the behaviour of small particles which have been done by quantum mechanics developed by Heisenberg, Schrodinger, Barn in 1925–1926 and Dirac in 1927 and other physicists.

4·6 Limitations of Newton's laws of motion

Limitations of Newton's laws of motion are discussed below briefly.

- Newton's laws of motion are applicable for large bodies. The particles whose masses are exceedingly small, viz. for electron, proton, neutron etc Newton's laws of motion are not applicable.
- For very small particles ($m \approx 10^{-31}$ kg) velocities are very large, i.e., almost close to the velocity of light, as a result these particles while in motion show wave behaviour. Newton's laws of motion are not applicable for these particles. For these particles theory of relativity is applicable.
- Further, when the acceleration of a body becomes very small ($<10^{-10}$ ms^{-2}), then good results are not obtained by applying Newton's laws of motion. In this case, force is proportional to the square of acceleration. Newton's laws of motion are applicable only for the cases where force is proportional to acceleration.
- When a body is in stationary frame of reference or in motion with uniform velocity then Newton's laws are applicable, otherwise will not be applicable.
- When the velocity of a body is very small compared to the speed of light then Newton's laws of motion can be applied. In case of bodies having speed close to the velocity of light Newton's laws cannot be applied. In this case Einstein's theory of relativity is applied.

4·7 Concept of force, field and intensity

In the previous section, idea has been given about force and its different forms. We have learnt that, **"The external cause that changes the static body into motion or changes the motion of a body or tends to change the state of a body is called force."** Force is a vector quantity.

Nature of force : Gravitational force is the natural attractive force between two bodies. Two charged bodies attract each other when the two charges are of opposite nature i.e., one is positive and the other one is negative and repel each other when the two charges are similar. Gavitational force does not depend on the medium whereas electric force depends on medium.

Field : The influence of a charge is observed around a large area of the charge. If another charge is brought in that region, it experiences force. Again, the second charge exerts force on the first charge. That means, the force acting between the two charges are mutual. Now, if the amount of charge increases, force will also increase; and if the distance between the two charges is increased, then force between them decreases.

Similarly, around a body there exists its influence. If another body is brought in that region, then that body experiences force. This force is called gravitational force. This force mutually acts between the two bodies. Now, if the mass of the body increases, then force will also increase. Again if distance between the two bodies increases then force decreases.

In above discussion it is observed that the force acting between the two bodies or between the two charges acts at a distance without any connection or contact between them. But question arises how the force acts without any physical contact between them. Famous scientist **Michael Faraday** first realised that around a charge agitation is produced; consequently any charge placed in that region will experience a force. He named this agitation as **electric field.** So electric field can be defined as follows :

Difinition : The region around a charge in which it exerts its influence is called electric field.

In discussion relating gravitation concept of similar field is introduced. According to this idea "**the region around a body in which its attractive force is experienced that region is called gravitatioal field of that body.**" So, gravitational field acts as the medium for the propagation of gravitational force.

Field Intensity : All over the electric or gravitational field its influence is not same. The force that a charge experiences near a charged body will be less at long distance. Again, if the amount of charge is more, then it will experience more force. This weakness or strength of the electric field is expressed by an electric quantity. It is called intensity of the electric field or electric intensity.

Definition : The force acting on a unit charge placed at a point in an electric field is the electric field intensity or electric intensity.

Now, if $\vec{F}$ is the electric force and q_0 is the charge, then according to the definition

electric field $\vec{E} = \dfrac{\vec{F}}{q_0}$ or, $\vec{F} = q_0\vec{E}$.

It is a vector quantity. Its unit is NC^{-1}.

Similarly, force in all points in the gravitational field is not same. That means, the gravitational field intensity is different. In order to determine the intensity of gravitational field at a point, a body of unit mass is considered at that point. The force experienced by that unit mass is measured as the gravitational field intensity.

Definition : The amount of force experienced by a unit mass placed at a point in gravitational field is called the gravitational intensity at that point due to that field.

Now, if electric force is $\vec{F}$ and charge is q_0, then according to the definition, $\vec{E} = \frac{\vec{F}}{q_0}$ or, $\vec{F} = q_0 \vec{E}$. It is a vector quantity. Its unit is NC^{-1}.

Similarly, this force is not active at all points of the gravitational field. That means, gravitational field becomes different. In order to determine the intensity at a point in a gravitational field, a body of unit mass is considered at that point. The force that is experienced by the unit mass is the measure of the gravitational field intensity.

Definition : If a body of unit mass is placed at a point of gravitational field, the force that is applied on that mass is called the gravitational intensity at that point due to that field.

So, if a force $\vec{F}$ acts on a body of mass m placed at a point in the gravitational field, then gravitational field intensity at that point will be,

$$\vec{E} = \frac{\vec{F}}{m} \quad \cdots \quad \cdots \quad \cdots \quad (4.12)$$

Intensity has both magnitude as well as direction. The direction of the intensity indicates the direction of gravitational field. Its unit is Nkg^{-1}.

4·8 Conservation of linear momentum

Principle of conservation of linear momentum is an important matter in physics. From Newton's law this principle is obtained. Some bodies can exert force (action-reaction) on one another and due to its influence the body may be in motion, but if no external force is applied, their total momentum remains unchanged. You have noticed that a person sitting on a chair cannot pull that chair by applying force on it. Can you explain the reason behind it ? Since the chair and the person are at rest, so total momentum is zero. Now if the person tries to pull the chair up i.e., apply upward force on the chair, the chair will exert equal downward reaction force on the person. These two forces are the acting force between the chair and the person. Since there is no external force acting on them, so total momentum of the chair and the person will remain zero. So, the chair will not rise.

Similarly, a person sitting on a chair cannot pull himself up by pulling his hair up by hand. Again, if a car stops, passengers inside the car cannot move the car even though they push the car from inside. Answers of these phenomena is available from the principle of conservation of linear momentum.

From Newton's third law of motion we can know about the conservation principle of momentum. Principle of conservation of momentum is equally applicable for all bodies— small or large, earthly or terrestrial. From Newton's first law of motion we know that if net applied force on a body is zero, then a moving body will continue its motion with uniform speed along a straight line i.e., its speed remains constant. If velocity $\vec{v}$ is constant with respect to time then momentum $\vec{p} = m\vec{v}$ and remains constant with respect to time.

4·9 Conservation principle of linear motion or conservation principle

From Newton's second law of motion we know that the rate of change of mementum of a body is proportional to the force applied on it. So, if external force is not applied, then there will not be any change of momentum. That means, linear momentum of that body will remain unchanged. This is **the conservation principle of linear momentum or conservation principle.**

Principle : If external force is not applied on a body, then momentum will not change. That means, momentum remains conserved.

Explanation : Suppose two bodies of mass of m_1 and m_2 while moving with velocities u_1 and u_2 respectively are collided. After collsion the two bodies move with velocities v_1 and v_2 respectively along the same strainght line [Fig. 4·11].

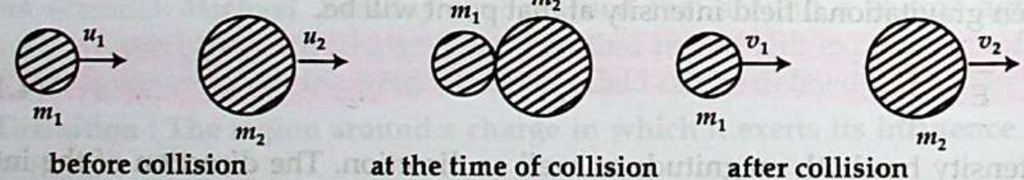


Fig. 4·11

So, total momentum of the two bodies before collision = $m_1u_1 + m_2u_2$

and their momentum after collision = $m_1v_1 + m_2v_2$

Now, if no external force is applied then according to the conservation principle of momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \ldots \quad \ldots \quad \ldots \quad (4.13)$$

So, total linear momentum is conserved or remains unchanged.

4·9·1 Example of conservation principle of linear momentum or Conservation principle

Look at the following examples and you will be able to explain how conservation principle of linear momentum is effective there.

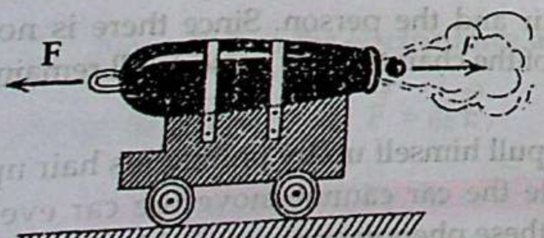


Fig. 4·12

Example 1. When a bullet is fired from a cannon, it goes ahead with tremendous velocity. Before firing both the cannon and the bullet were at rest, so momentum was zero. But after firing the cannon the bullet acquires a momentum. The cannon acquires equal but oppositely directed momentum. Because of it the cannon gets force towards back i.e., it recoils back [Fig. 4·12].

Example 2. When a passenger jumps from a boat to the bank, the boat pushes back. Before jump the boat and the passenger were at rest, so their total momentum was zero. As the passenger jumps, he becomes mobile and acquires momentum. According to the principle of conservation of momentum, total momentum is zero. So, there occurs equal and oppositely directed momentum in the boat. Hence the boat being mobile pushes back [Fig. 4·13].

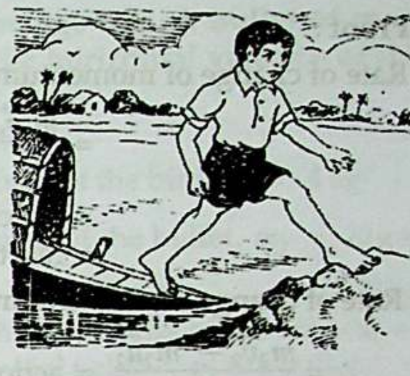


Fig. 4·13

Work : Explain the reason of drifting of a boat while jumping from the boat.

Example 3. Due to combustion of fuel, generated gas rushes out through the orifice

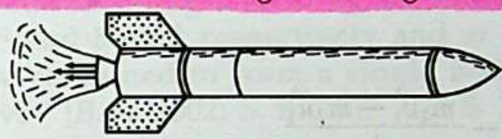


Fig. 4·14

at the back side of a rocket or jet plane and hence the rocket or plane moves ahead with equal momentum [Fig. 4·14].

4·10 Verification of conservation principle

Mathematical Method : Mathematically, the principle of conservation of momentum can be verified.

Suppose two particles of masses m_1 and m_2 are moving along same direction in a straight line with velocities $\vec{u}_1$ and $\vec{u}_2$ respectively [Fig. 4·15]. Here $\vec{u}_1 > \vec{u}_2$. At one time the first particle strikes the second particle from behind and afterwards the two particles continue to move in the same direction in a straight line with velocities $\vec{v}_1$ and $\vec{v}_2$ respectively.

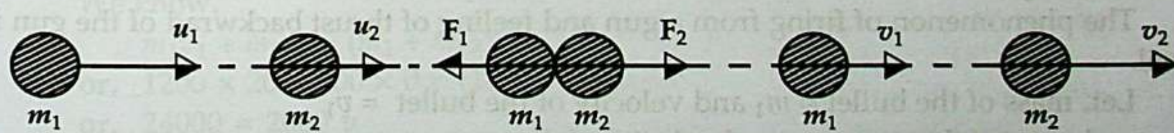


Fig. 4·15

Let the time of collision of action and reaction be t. Then

Summation of the initial momentum of the two particles $= m_1\vec{u}_1 + m_2\vec{u}_2$

Summation of the final momentum of the two particles $= m_1\vec{v}_1 + m_2\vec{v}_2$

From the laws of conservation of momentum it is to be proved that,

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

Proof :

Rate of change of momentum of the first particle is

$$= \frac{m_1\vec{v}_1 - m_1\vec{u}_1}{t}$$

$= $ reaction force $= \vec{F}_1$

= reaction force of the second particle on the first particle

Rate of change of momentum of the second particle

$$= \frac{m_2\vec{v}_2 - m_2\vec{u}_2}{t} = \text{action force} = \vec{F}_2$$

= applied force of the first particle on the second particle.

But rate of change of momentum of the two bodies (i.e., action and reaction forces) is equal and opposite.

i.e., $\vec{F}_2 = -\vec{F}_1$

$$\therefore \quad \frac{m_2\vec{v}_2 - m_2\vec{u}_2}{t} = -\frac{m_1\vec{v}_1 - m_1\vec{u}_1}{t}$$

or, $m_2\vec{v}_2 - m_2\vec{u}_2 = -m_1\vec{v}_1 + m_1\vec{u}_1$

or, $m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$ .... = a constant vector

$\therefore$ sum of the initial momentum of the two particles = sum of the final momentum of the two particles.

i.e., $\Sigma m\vec{v} =$ constant vector ... ... (4.14)

So, due to the action and reaction forces no change of total momentum takes place, amount of momentum that a body loses, the other body acquires exactly the same amount of momentum. That means, momentum remains same before and after the collision. Hence, the principle of conservation of momentum is proved.

Recoil of a gun

The phenomenon of firing from a gun and feeling of thrust backwrad of the gun is recoil.

Let, mass of the bullet $= m_1$ and velocity of the bullet $= v_1$
and mass of the gun $= m_2$ and velocity of the gun $= v_2$
their total momentum before firing = 0 and their total momentum after firing

$= m_1v_1 + m_2v_2$

From conservation principle of momentum, we get

$$0 = m_1v_1 + m_2v_2$$

or, $v_2 = -\frac{m_1}{m_2}v_1$ ... ... (i)

This velocity v_2 is the recoil or recoil velocity. By the negative sign of equation (i) it is understood that v_1 and v_2 are oppositely directed. That means, the direction along which a bullet emerges, the gun moves in the opposite direction.

Mathematical examples

1. A bird having mass of 4 kg is sitting in a mango tree. A bullet of mass 20 g struck the bird at a velocity of 200 ms^{-1}. Calculate the horizontal velocity of the bird if the bullet remains inside the bird.

Here,
mass of the bird, $m_1 = 4$ kg
mass of the bullet, $m_2 = 20 \text{ g} = \frac{20}{1000}$ kg $= 0.02$ kg
initial velocity of the bird, $u_1 = 0$
initial velocity of the bullet, $u_2 = 200$ ms^{-1}
final velocity of the bird, $v_1 = ?$
final velocity of the bullet, $v_2 = 0$

We know,
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
or, $4 \times 0 + 0.02 \times 200 = 4 \times v_1 + 0.02 \times 0$
or, $0 + 4 = 4v_1 + 0$
or, $4v_1 = 4$
$\therefore$ $v_1 = 1$ ms^{-1}

2. Two bodies of masses 40 kg and 60 kg are moving opposite to each other with velocities 10 ms^{-1} and 5 ms^{-1} respectively and at a time they collide. After collision the bodies are combined to form a single body. With what velocity the combined body will move ? [B. B. 2002; S. B. 2002; Ch. B. 2001; J. B. 2000; R. B. 2001]

Here,
$m_1 = 40$ kg
$m_2 = 60$ kg
$u_1 = 10$ ms^{-1}
$u_2 = -5$ ms^{-1}
[$\because u_1$ and u_2 are opposite]
$v_1 = v_2 = v = ?$

Let the velocity of the first body be +ve, then the velocity of the second body be $-$ve.
We know, $m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$
Let the velocity of the combined body be v.
i.e., if $v_1 = v_2 = v$, then $m_1v_1 + m_2v_2 = v(m_1 + m_2)$
$40 \times v + 60v = 40 \times 10 + 60 \times (-5)$
or, $100v = 400 - 300$
or, $100v = 100$
$\therefore$ $v = 1$ ms^{-1}

3. A car of mass of 1200 kg was moving with velocity of 20 ms^{-1}. While moving it collided with a car of mass of 800 kg at rest. After collision the cars combined and moved 50 m ahead and stopped. What was the magnitude of the resisting force.

Here,
mass of the first body, $m_1 = 1200$ kg
mass of the 2nd body, $m_2 = 800$ kg
velocity of the first body, $v_1 = 20$ ms^{-1}
velocity of the 2nd body, $v_2 = 0$
combined velocity, $v = ?$
final velocity, $v' = 0$
distance, $s = 50$ m
resisting force, $F = ?$

We know,
$$m_1v_1 + m_2v_2 = (m_1 + m_2)\,v$$
or, $1200 \times 20 + 800 \times 0 = (1200 + 800)\,v$
or, $24000 = 2000\,v$
$\therefore$ $v = 12$ ms^{-1}
Again, $v^2 = v_0^2 + 2as$
or, $0 = (12)^2 + 2 \times a \times 50$
or, $0 = 144 + 100\,a$
or, $100\,a = -144$
$\therefore$ $a = -1.44$ ms^{-2}

$\therefore$ Resisting force, $F = ma = 2000 \times -1.44 = -2880$ N

4. A bullet of mass 0·01 kg is released from a gun of mass 6 kg with a velocity of 300 ms^{-1}. Calculate the recoil velocity of the gun.

Here, $M = 6$ kg, $m = 0·01$ kg, $v = 300\ ms^{-1}$, $V = ?$

Let the recoil velocity of the gun = V

According to the conservation principle of momentum we get,

$$MV + mv = 0 \quad \ldots \quad \ldots \quad (i)$$

or, $MV = -mv$

$\therefore$ from equation (i) we get, $V = \frac{-mv}{M} = \frac{0·01\ kg \times 300\ ms^{-1}}{6\ kg} = 0·5\ ms^{-1}$

5. The recoil velocity of a gun of mass of 8 kg is 10 ms^{-1} when a bullet of mass of 10 g leaves from the gun. After penetrating 0·3 m inside the target the bullet stops. Calculate the applied resistance on the bullet.

Since before firing the bullet and the gun were at rest, so the magnitude of their total momentum = 0.

Now, if m_1 and m_2 are the masses respectively of the gun and the bullet and if v_1 and v_2 are their respective velocities, then from conservation principle of momentum we get,

$$0 = m_1v_1 + m_2v_2 \quad \ldots \quad \ldots \quad (i)$$

or, $0 = 8 \times 10 + \frac{10}{1000} \times v_2 = 80 + 1 \times 10^{-2}\ v_2$

or, $v_2 = -\frac{80}{1 \times 10^{-2}} = -8 \times 10^3\ ms^{-1}$

After penetrating 0·3 m inside the target the velocity of the bullet becomes zero. If the retardation of the bullet is a, from equation $v^2 = u^2 - 2as$ we get,

Here, $u = -8 \times 10^3\ ms^{-1}$, $m = 10\ g = 0·010$ kg

$0 = (-8 \times 10^3)^2 - 2a \times 0·3$

or, $a = \frac{(-8 \times 10^3)^2}{0·6} = \frac{64 \times 10^6}{0·6}$

$= 1·067 \times 10^8\ ms^{-2}$

$\therefore$ resistance, $P = ma = 0·010 \times 1·067 \times 10^8 = 1·067 \times 10^6$ N

4·11 Newton's third law of motion and conservation of momentum

Newton's third law is nothing but action and reaction. When a body exerts force on another body, then the second body also exerts equal and oppositely directed force on the first body. The force that the first body exerts on the second body is considered as **action** and the force applied by the second body on the first body is called **reaction.**

Whether the two bodies are at rest or at motion, or one touches the other or not or stay apart from each other Newton's third law applies in all cases.

The relation of action and reaction is not causal relation. Action and reaction do not act one after another. Two forces act simultaneously. As long an action acts reaction force also acts for the same duration. When action stops reaction also stops.

In nature forces always act in pairs. There is nothing called single isolated force. When we say that one force is active, actually we say one force instead of two active forces. These two forces are complementary to each other.

From the above discussion we get an idea about Newton's third law of motion. The law is :

To every action there is an equal and opposite reaction i.e., for every acting force there is an equal and opposite reacting force. This law may be called mutual action of forces between the bodies. So, if the acting force is $\vec{F}$ and the reacting force is $\vec{R}$, then $\vec{F} = -\vec{R}$. **On the otherhand, if no force other than action and reaction are active, then no change of linear momentum takes place.**

Explanation : According to Newton's third law if a body A applies force on another body B, then the body B will also exert equal and opposite force on A [Fig. 4·16].

Force applied by A is **action** and force applied by B is **reaction**. So, if action is $\vec{F}$ and reaction is $\vec{R}$, then $\vec{F} = -\vec{R}$

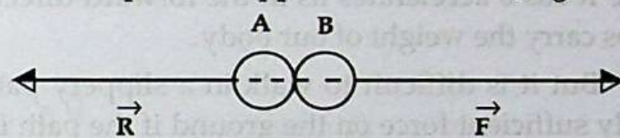


Fig. 4·16

Action and reaction act on two different bodies. If there is no action, there will be no reaction. If the duration of action or reaction is t, then,

$$\vec{F} \times t = -\vec{R} \times t \quad \ldots \quad \ldots \quad \ldots \quad (4.15)$$

That means, **impulse of force of action = − impulse of force of reaction.**

It is equally applicable to bodies at rest or in motion.

Newton's third law of motion and principle of conservation of momentum are explained by some examples.

Examples :

1. A book on a table : If a book is kept on a table, weight of the book will exert pressure perpendicularly on the table. It is action. According to Newton's third law of motion the table will also exert upward force on the book. It is reaction. Since action and reaction are equal and opposite, so the book will remain in stable position on the table.

2. To fire from a gun : When a hunter fires a shot from the gun, he feels a backward thrust. Initially velocity of both the gun and the bullet is zero. As a result their combined momentum remains zero. When a bullet is fired, it acquires momentum in the forward direction. According to Newton's third law of motion the gun also acquires an equal and opposite momentum i.e., the gun will go backward with same momentum and the hunter will feel backward thrust.

3. To jump from a boat : When a passenger jumps from a boat on the bank of a river, then the boat is seen to go backward. The boat goes backward because of the force applied by the passenger on the boat. According to Newton's third law the boat also exerts equal and oppositely directed force on the passenger. As a result the passenger reaches the shore.

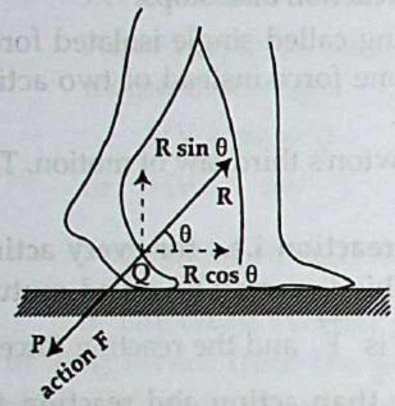


Fig. 4.17

4. Walking : When we walk, the front foot exerts a normal downward force on the ground. This is the action force. The ground in turn exerts an equal and opposite force on the front foot. It is the reaction force. Now since the action and reaction are equal, so the front foot remains stationary. But the back foot exerts a force $\vec{F}$ obliquely on the ground at point Q along QP [Fig. 4.17]. This force makes an angle θ with the horizontal. According to Newton's third law the ground below the foot exerts equal and opposite force on the foot. Let this reaction be $\vec{R}$. So $\vec{R} = -\vec{F}$. The horizontal component of the reaction force R cos θ accelerates us in the forward direction and the vertical component R sin θ helps carry the weight of our body.

But it is difficult to walk in a slippery path. Because it is difficult for the foot to apply sufficient force on the ground if the path is slippery. As a result the reaction force of the ground on the foot and also along with it the horizontal component of reaction force become less. For this reason it is difficult to walk on a slippery road. Similar problem arises while walking on a marble floor and a sandy road.

Work : Why the tyres of a car are grooved on outer surface ?

Tyres of a car are made with grooved on outer surface. This is because that necessary frictional force are acquired by the car. Due to these grooves the tyres can grab the road correctly. If it could not grab the road the car would not have been able to move. Besides, during motion if brake would have applied the car could have skidded. So, in order to drive the car properly grooves are cut on the outside surface of a tyre.

Work : When a balloon filled with air is released by opening the mouth, why it runs in the opposite side of the open mouth ?

When a balloon is contracted a force is applied on air in it. So air rushes through the open mouth. According to Newton's third law, this time air also applies opposite force on the balloon. So, if air-filled balloon is released by opening the mouth, it is seen to run in the opposite side of the open mouth.

4.12 Newton's third law and mathematical explanation of conservation of momentum

We know from Newton's first law of motion that if the applied force on a body is zero, then the body will move along a straight line with uniform velocity. If the velocity

$\vec{v}$ remains constant with respect to time, then momentum $\left(\vec{P} = m\vec{v}\right)$ will also remain constant with time.

Law : When the external force applied on a system is zero, then total momentum of the system remains conserved.

Suppose there are two bodies of masses m_1 and m_2. No external force is acting on these bodies. So, the two bodies are moving only due to action and reaction forces. If applied force on m_1 by m_2 is F_1, then according to Newton's third law equal and opposite force F_2 will act on m_2 by m_1. That means,

$$F_1 = -F_2 \qquad \ldots \qquad \ldots \qquad \ldots \quad (4.16)$$

Action and reaction forces act at the same time.

Let the momentum of the two bodies of masses m_1 and m_2 be P_1 and P_2, respectively. So, according to Newton's second law of motion,

$$F_1 = \frac{dP_1}{dt} \text{ and } F_2 = \frac{dP_2}{dt}$$

∴ From equation (4.15) we get,

$$\frac{dP_1}{dt} = -\frac{dP_2}{dt} \quad \text{or,} \quad \frac{dP_1}{dt} + \frac{dP_2}{dt} = 0$$

$$\text{or,} \quad \frac{d}{dt}(P_1 + P_2) = 0 \qquad \therefore \quad P_1 + P_2 = \text{constant.}$$

That means, if no external force is applied, then momentum remains constant. **It is the conservation principle of momentum.**

From the above discussion we have learnt the following things :

(1) While deriving the principle, the nature of action and reaction forces have not been discussed.

(2) This principle is applicable for any type of mutual actions.

(3) Momentum is a vector quantity. That means according to this principle change of momentum of isolated combined bodies can only be possible by external force.

(4) By using this principle, complicated problems about the mutual actions of more than one body can be solved.

Practical Work : Sitting on a rickshaw you ask the rickshaw puller to start the rickshaw. The rickshaw will move. Now when the rickshaw moves from a plain road to a higher road, then the speed will be reduced. Now you stand up from your seat and lean your body forward and apply force on the seat of the rickshaw. The rickshaw will move faster than before, but why ? Explain.

As the road becomes higher, so speed of the rickshaw decreases, hence momentum also decreases. Again, due to the application of force on the rickshaw momentum will be generated. Consequently, the rickshaw will move ahead. But total momentum will remain conserved.

4·13 Rotational motion

With the change of time if the position of a body is changed then its state is said to be in motion. For example—car, man etc. If a moving body moves in a straight line then the motion of that body is called **translational motion.** If a body is released from the top of a building or a car moving in a straight line the motion is called translational motion.

Again, if a body is in motion around a circular path centring a fixed point or axis, then that motion is called **rotational motion.** Example—motion of an electric fan. The axis around which it rotates is called the axis of rotation.

Characteristics of rotation

Rotational motion has the following characteristics—

(i) If a body rotates then each particle in it rotates through the same angle in a particular interval of time.

(ii) Rotational axis always remains stationary.

4·14 Terms related to angular momentum

4·14·1 Angular displacement

Suppose, a particle is rotating around a fixed point O on a plane like this page of the book is circular path. Here the axis of rotation will pass through the centre O and will be normal to the plane of the circle [Fig. 4·18]. In order to know the position of the particle at any point a fixed straight line OX is imagined on that plane. OX is called the **reference line.**

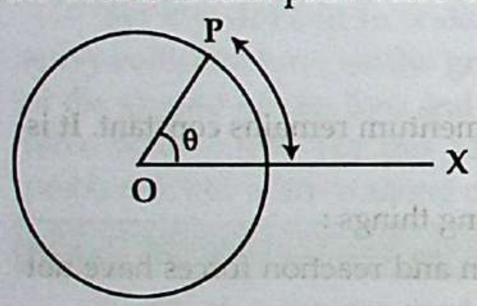


Fig. 4·18

From the moment the particle crosses the reference line, let us start to count the time and let after time t the position of the particle is P. Obviously if the angle between the radius OP and the line OX is known then the position of the particle will be known completely. Angle θ is called the **angular displacement** of the particle. OP is a radius vector.

Definition : The angle through which a radius vector of a particle moving in circular motion is displaced at a particular time interval is called the angular displacement for that particular time interval.

If it is expressed in radian then the relation between angular displacement θ and associated arc s becomes very simple. If r is the radius vector, then it can be written,

$$\theta = \frac{s}{r} \quad \ldots \quad \ldots \quad \ldots \quad (4.17)$$

4·14·2 Angular velocity

Like linear motion angular motion can also be uniform or non-uniform (accelerated). **If the angular motion is non-uniform then the ratio of the angular displacement and the time interval for that displacement is called the average angular velocity. It is** denoted by ω. Anglular velocity can be positive or negative. Like angular displacement similar rule is followed here.

If in exeedingly small interval of time Δt the angular displacement of a particle is $\Delta\theta$ [Fig. 4·19], then average angular velocity during the time interval will be,

$$\omega = \frac{\Delta\theta}{\Delta t} \quad \ldots \quad \ldots \quad \ldots \quad (4.18)$$

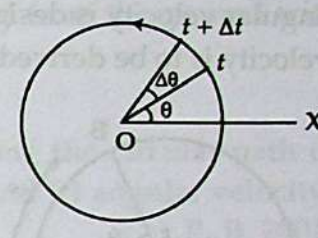


Fig. 4·19

In order to know the angular velocity at a particular instant it is needed to make time interval small to smaller. If the limiting value of time interval is zero, then during that very small time interval average angular velocity is equal to instantaneous angular velocity. So in a very small time the rate of change of angular displacement is called instantaneous angular velocity ω.

That means,

$$\omega = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Normally, angular velocity means the instantaneous velocity.

If the magnitude of the angular velocity remains constant then circular motion is called uniform circular motion. In case of uniform circular motion if θ is the angular displacement in time t then magnitude of angular velocity becomes,

$$\omega = \frac{\theta}{t} \text{ or, } \theta = \omega t \quad \ldots \quad \ldots \quad \ldots \quad (4.19)$$

This equation is similar to the equation of uniform linear motion, $s = vt$.

Unit : Generally angular velocity is expressed in the unit of radian/second or simply rad/s. In mechanical engineering another unit is used. Its name is revolution per minute or simply rpm.

Dimension of angular velocity : $|\omega| = \left|\frac{\text{linear velocity}}{\text{radius}}\right| = \frac{[LT^{-1}]}{[L]} = [T^{-1}]$

Time taken by the particle to complete one revolution is called **time period**. One complete revolution means 2π radian of angular displacement. So, if time period is T, then according to equation (4·19),

$$\omega = \frac{2\pi}{T} \quad \ldots \quad \ldots \quad \ldots \quad (4.20)$$

In $\frac{1}{T}$ unit time means total number of revolution. It is called frequency. If it is denoted by n then we get, $\omega = 2\pi n$.

Again, if time is t and number of revolution is N, then $\omega = \frac{2\pi N}{t}$

$$\therefore \quad \omega = 2\pi n = \frac{2\pi}{T} = \frac{2\pi N}{t} \quad \ldots \quad \ldots \quad \ldots \quad (4.21)$$

4·15 Relation between angular velocity and linear velocity

We know, linear displacement per second of a body in a linear path at particular direction is called linear velocity and angular displacement per second of a body in a circular path is called angular velocity. Linear velocity is designated either by v or v_0 and angular velocity is designated as ω. Now an equation relating linear velocity and angular velocity is to be derived.

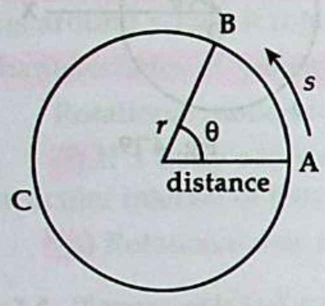


Fig. 4·20

Suppose a particle is rotating along the circumference of a circle if radius r with uniform angular velocity ω [Fig. 4·20]. If in T second the particle rotates the circumference of the circle once, then according to the definition of angular velocity.

$$\omega = \frac{\text{angular distance}}{\text{time}} = \frac{2\pi}{T}$$

$$\text{or, } T = \frac{2\pi}{\omega} \quad \ldots \quad \ldots \quad \ldots \quad (4.22)$$

Again, if angular velocity is ω and angular displacement is θ, then number of rotation, $N = \frac{\theta}{2\pi}$

Now, instead of rotating along the circular path if the particle travels the distance equal to the circumference of the circle in straigat line in time T, then

$$v = \frac{\text{circumference of the circle}}{\text{time to travel the distance equal to the circumference}} = \frac{2\pi r}{T}$$

$$\text{or, } T = \frac{2\pi r}{v} \quad \ldots \quad \ldots \quad \ldots \quad (4.23)$$

From equations (4.21) and (4.22), we get $T = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$

$$\text{or, } \frac{1}{\omega} = \frac{r}{v}$$

$$\text{or, } v = \omega r \quad \ldots \quad \ldots \quad (4.24)$$

That means, linear velocity = angular velocity × radius of the circle.

Vector form of the equation (4.24) is : $\vec{v} = \vec{\omega} \times \vec{r}$; directions of these three vectors ar4e shown in fig. 4.21.

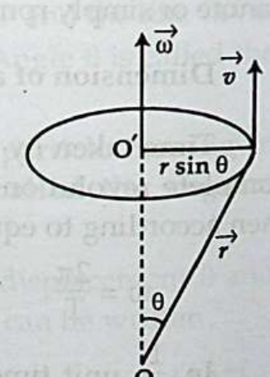


Fig. 4·21

It is to be mentioned that if the circular motion is non-uniform even then at any point $v = \omega r$. If the body moves with uniform angular velocity, then ω = constant. So $v \propto r$ i.e., linear velocity is proportional to the distance from the rotational axis.

Example : The distant cow in a husking mill of paddy needs to work with maximum velocity.

Inquisitive work : Sometimes ejected cricket ball by a baller is reflected with larger velocity from the ground than the velocity of ejection—Explain.

During touching the ground if there is spin or rotation of the cricket ball, then spin or rotational kinetic energy of the ball will be added to the linear kinetic energy. As a result, due to the combined kinetic energies the ball is reflected from the ground with larger velocity than the velocity of ejection of the ball.

Mathematical examples

1. A particle is executes 120 revolutions per minute along the circular path of radius 1·5 m. What are its (a) linear velocity, (b) time period and (c) angular velocity? [R. B. 2001]

Here,
radius of the circular path, $r = 1{\cdot}5$ m
number of revolution or frequency,

$$n = \frac{120}{1\text{ min}} = \frac{120}{60\text{ s}} = 2\text{ s}^{-1} = 2\text{ Hz}$$

We know,
(a) linear velocity, $v = \omega r$

$$\therefore\ v = 2\pi n r = 2 \times 3{\cdot}142 \times 2 \times 1{\cdot}5 = 18{\cdot}852\text{ ms}^{-1}$$

(b) time period, $T = \frac{t}{N} = \frac{60}{120} = 0{\cdot}5$ s $\left[\because T = \frac{1}{n} = \frac{1}{\frac{N}{t}} = \frac{t}{N}\right]$

or, $T = \frac{1}{n} = \frac{1}{2} = 0{\cdot}5$ s

(c) angular velocity, $\omega = 2\pi n = \frac{2\pi}{T} = \frac{2 \times 3{\cdot}142}{0{\cdot}5} = 12{\cdot}568\text{ rad s}^{-1}$

Ans. (a) $18{\cdot}852\text{ ms}^{-1}$, (b) 0·5 s, (c) $12{\cdot}568\text{ rad s}^{-1}$

2. What is the angular velocity of the minute hand of a wrist watch ?

Here,
angular displacement, $\theta = 2\pi$
$t = 60\text{ min} = 60 \times 60 = 3600$ s

We know,

$$\omega = \frac{\theta}{t} = \frac{2\pi}{3600} = \frac{\pi}{1800}\text{ rad s}^{-1}$$

4·16 Angular acceleration

In many cases angular velocity of a rotating particle increases or decreases. If angular velocity changes then it is understood that the particle is moving with angular acceleration.

Average angular acceleration of a rotating particle means rate of change of angular velocity for a fixed time interval.

So, if for a very small time interval Δt the change of angular velocity is $\Delta\omega$, then average angular acceleration during that time interval is,

$$\vec{\alpha} = \frac{\Delta\omega}{\Delta t} \quad \ldots \quad \ldots \quad \ldots \quad (4.25)$$

So the definition of angular acceleration can be given as follows :

Definition : If time interval tends to zero, then the rate of change of angular velocity of a body with respect to time is called instantaneous angular acceleration.

By applying the rule of calculus, we get,

$$\alpha = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta\omega}{\Delta t}$$

$$\text{or,} \quad \alpha = \frac{d\omega}{dt} = \frac{d}{dt}\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} \quad \ldots \quad \ldots \quad \ldots \quad (4.26)$$

Angular acceleration means instantaneous angular acceleration.

Its unit is rad/sec^2 (rad s^{-2}).

If the angular acceleration of a rotating particle is constant, then instantaneous angular acceleration in any time interval is equal to the average angular acceleration. In this case, if rate of change of angular velocity in time t is ω, then angular acceleration, $\alpha = \frac{\omega}{t}$

Dimension of angular acceleration, $[\alpha] = \left[\frac{\omega}{T}\right] = \left[\frac{T^{-1}}{T}\right] = [T^{-2}]$

4.17 Relation between angular accelaration and linear accelaration

Suppose a particle is rotating along the circumference of a circle of radius r [Fig. 4.22] with nonuniform motion. Let the linear velocity of the particle at time $t = v$, angular velocity = ω, linear acceleration = a and angular acceleration = α.

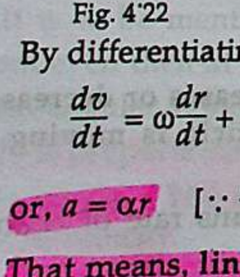


Fig. 4.22

We know,

$$v = \omega r \quad \ldots \quad \ldots \quad \ldots \quad (4.27)$$

$$\alpha = \frac{d\omega}{dt}$$

$$\text{and} \quad a = \frac{dv}{dt}$$

By differentiating equation (4.27) with respect to t, we get,

$$\frac{dv}{dt} = \omega\frac{dr}{dt} + \frac{d\omega}{dt}r = \frac{r\,d\omega}{dt} \quad [\because r = \text{constant}]$$

$$\text{or, } a = \alpha r \quad [\because \frac{d\omega}{dt} = \alpha]$$

That means, linear acceleration = angular acceleration × radius.

Mathematical examples

1. Distance between the moon and the earth is 3.84×10^5 km and the moon is rotating in a circular orbit around the earth and completes one rotation in 27.3 days. Calculate the angular and linear speed of the moon.

We know,

$$\omega = \frac{2\pi}{T} \text{ and } v = r\omega$$

$$\therefore \omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{27.3 \times 24 \times 60 \times 60}$$

$$= 2.662 \times 10^{-6} \text{ rad s}^{-1}$$

and $v_T = r\omega = 3.84 \times 10^5 \times 2.662 \times 10^{-6}$

$= 1.022$ kms^{-1}

Here,

$T = 27.3 \text{ days} = 27.3 \times 24 \times 60 \times 60$ s

$r = 3.84 \times 10^5$ km

$\omega = ?$

$v_T = ?$

2. An electric fan revolves 1500 times per minute. The fan stops in 4 minutes after switching off. What is the angular acceleration ? How many times will the fan revolve before stoppage ? [Ch. B. 2007]

We know,

$$\omega = \omega_0 + \alpha t$$

$$\text{or, } \alpha = \frac{\omega - \omega_0}{t} = \frac{0 - 50\pi \text{ rad s}^{-1}}{240 \text{ s}}$$

$$= -0.654 \text{ rad s}^{-2}$$

$$\text{Again, } \theta = \left(\frac{\omega_0 + \omega}{2}\right) t$$

$$\text{or, } \theta = \left(\frac{50\pi \text{ rad s}^{-1} + 0}{2}\right) \times 240 \text{ s}$$

$$= 6000\pi \text{ rad}$$

Here,

Initial angular velocity,

$$\omega_0 = 1500 \text{ rev}^{-1} \text{ min.} = \frac{1500 \times 2\pi \text{ rad}}{60 \text{ s}} = 50\pi \text{ rad s}^{-1}$$

Time, $t = 4$ minutes $= 4 \times 60$ s $= 240$ s

Final angular velocity, $\omega = 0$

Angular acceleration, $\alpha = ?$

Angular displacement, $\theta = ?$

$\therefore$ number of revolution before the stoppage of the fan $= \frac{6000\pi}{2\pi} = 3000$ rev.

Ans. — 0.654 rads^{-2}; 3000 rev.

4.18 Angular momentum

Definition : Vector product of radius vector of a rotating particle and the linear momentum is called the angular momentum.

Explanation : Suppose $\vec{r}$ = vector radius of a particle with respect to the centre of rotation and $\vec{P}$ = linear momentum of the body.

So, according to the definition, angular momentum of the body,

$$\vec{L} = \vec{r} \times \vec{P} = \hat{\eta} P \sin\theta \quad \dots \quad \dots \quad \dots \quad (4.28)$$

It is a vector quantity. Here $\hat{\eta}$ indicates the direction of product or the direction of angular momentum.

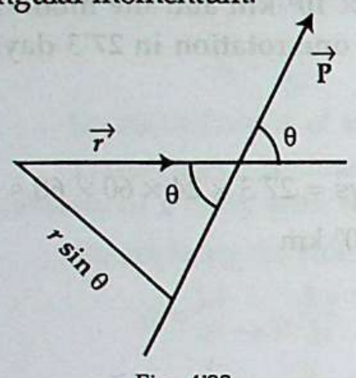


Fig. 4·23

Magnitude and direction : Magnitude of angular momentum, $L = rP \sin\theta$

Here θ is the angle between $\vec{r}$ and $\vec{P}$ [Fig. 4·23]. Perpendicular distance of the line of action of the momentum from the centre of rotation is $r \sin\theta$. So, product of the linear momentum of the particle and the perpendicular distance of the line of action from the axis of rotation gives the magnitude of the angular momentum.

Direction : The direction of $\vec{L}$ will be along the normal to the plane $\vec{r}$ and $\vec{P}$. This direction (direction of $\vec{L}$) is determined by the cross product rule.

Corollary : If the particle is moving in a circular path with respect to the centre of the circle, then angle θ between $\vec{r}$ and $\vec{P}$ is, θ = 90°, In that case,

$$L = rP\sin\theta = rP = r(mv) = mr\,(r\,\omega) = mr^2\omega \quad \ldots \quad \ldots \quad (4.29)$$

Unit and Dimensional Equation : In M.K.S. and S.I. systems unit of angular momentum is kgm^2s^{-1} and dimensional equation

$$[L] = [\text{momentum} \times \text{distance}] = [MLT^{-1}\,L] = [ML^2T^{-1}]$$

4·19 Relation between angular momentum and angular velocity

Let an object rotate about an axis with angular velocity ω. If the object is composed of many small particles, then we can write,

$L = l_1 + l_2 + l_3 + \ldots \ldots + l_n$, where, l_1, l_2, l_3, etc. are the angular momentum of individual particle.

Here l_1, l_2, l_3 etc. are parallel to one another.

Then,

$$\begin{aligned} L &= r_1p_1 + r_2p_2 + r_3p_3 + \ldots \ldots + r_np_n \\ &= r_1m_1v_1 + r_2m_2v_2 + r_3m_3v_3 + \ldots \ldots + r_nm_nv_n \\ &= r_1m_1\,(r_1\omega) + r_2m_2\,(r_2\omega) + r_3m_3\,(r_3\omega) + \ldots \ldots + r_nm(r_n\omega) \\ &= \omega\{m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots \ldots m_nr_n^2\} \\ &= \omega\,\Sigma\,mr^2 \quad [\because I = \Sigma\,mr^2] \\ &= I\omega \end{aligned}$$

$$\text{i.e., } L = I\omega \quad \ldots \quad \ldots \quad \ldots \quad (4.30)$$

Here $\omega = 2\pi n = \dfrac{2\pi}{T} = \dfrac{2\pi N}{t}$.

This is the relation between angular momentum and angular velocity. From equation (4.32) we can define angular momentum as follows.

Definition : The angular momentum of an object is the product of its moment of inertia about the axis of rotation and its angular velocity.

Vector form of angular momentum : Angular momentum is a vector quantity. The direction of the vector is along the axis of rotation. If a right-handed screw is rotated along the direction of rotation of the particle then the direction in which the screw advances is the direction of the angular momentum vector [Fig. 4·24].

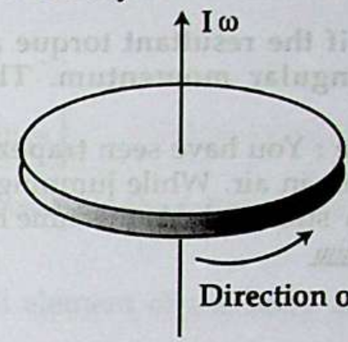


Fig. 4·24

Perceptional work : Show that the moment of inertia of a body rotating with uniform angular velocity is equal to its angular momentum.

We know, in case of rotational motion, angular momentum = moment of inertia × angular momentum = moment of inertia × angular velocity. or, $L = I\omega$.

If the angular velocity is units, or $\omega = I$, then $L = I$. So moment of inertia of a body rotating with unit angular velocity is equal to its angular momentum.

4·20 Law of conservation of angular momentum

From Newton's first law for angular motion we know that change of angular velocity i.e., angular momentum occurs due to the action of external torque only. If there is no torque, the body will rotate in uniform angular velocity. That means, angular velocity with respect to time becomes constant. As a result, angular momentum also becomes constant. This is called conservation principle of angular momentum. So it can be said that if **resultant of the torque on a body is zero, the angular momentum of the body remains conserved.**

Mathematical proof : We know, angular momentum,

$$L = I\omega \quad \ldots \quad \ldots \quad \ldots \quad (4.31)$$

Here L is the angular momentum of the body, I is the moment of inertia and ω is the angular velocity.

Differentiating equation (4·31) with respect to time,

$$\frac{dL}{dt} = \frac{d}{dt}(I\omega) = I\frac{d\omega}{dt}$$

But $\frac{d\omega}{dt} = \alpha$

So, $\frac{dL}{dt} = I\alpha = \tau$ [according to Newton's second law of angular motion]

Now, $\tau = 0$, i.e., if torque is not active on the body,

$$\frac{dL}{dt} = 0 \quad \therefore \quad L = \text{constant}$$

Hence, if the resultant torque acting on a body is zero, there will not be any change of angular momentum. This is the conservation principle of angular momentum.

Example : You have seen **trapeze** game in the circus. There players demonstrate many exercises in air. While jumping from a swinging pad the hands and feet of the player remain stretched. At this time his angular velocity is minimum. Now when hands and feet are folded and brought near to the chest, the angular velocity increases; so, it becomes easier for him to somersault in air successively. Due to folding of hands and feet moment of inertia (I) of the player becomes less, but as his angular momentum $L = I\omega$ remains constant so his angular velocity increases [Fig. 4·25].

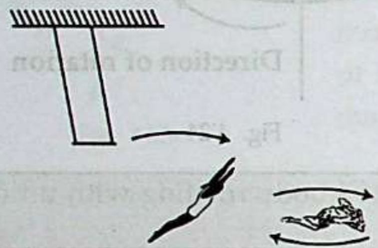


Fig. 4·25

Verify : Explain rotation of the body while jumping from the diving board or while skating on ice feat showing rotation on toes many times.

4·21 Moment of inertia and radius of gyration

4·21·1 Moment of inertia

When a rigid body is confined in a fixed axis, then if force is applied on that body, it cannot move in straight line due to confinement. The body rotates around the axis and there is angular displacement of each of the particles of the body. This type of motion of a body with respect to an axis is called rotational motion. The axis can either be inside or outside the body.

Definition : If a rigid body rotates around an axis, then moment of inertia of that body with respect to that axis means the summation of the product of square of distance from the axis and mass of each of the particles of that body.

Explanation : Let B be a rigid body which is rotating around a fixed axis XY with a uniform angular velocity ω [Fig. 4·26]. If the body is the summation of innumerable particles of masses m_1, m_2, m_3 .... m_n and the particles are respectively at distances r_1, r_2, r_3 .... r_n from the axis of rotation, then according to the definition with respect to that axis,

The moment of inertia of first particle = $m_1 r_1^2$

The moment of inertia of second particle = $m_2 r_2^2$

The moment of inertia of third particle = $m_3 r_3^2$ and

The moment of inertia of n-th particle = $m_n r_n^2$

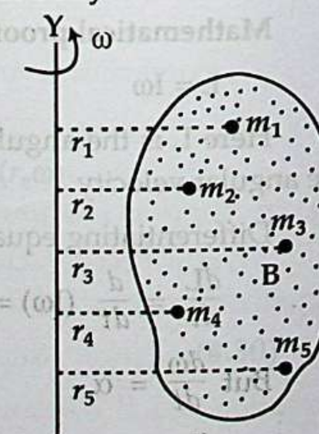


Fig. 4·26

So, the moment of inertia for the whole body with respect of that axis is,

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots\ldots + m_nr_n^2$$

$$\therefore\ I = \sum_{i=1}^{n} m_ir_i^2 \qquad \ldots \qquad \ldots \qquad (4.32)$$

$\left[\sum_{i=1}^{n}\right.$ sign indicates summation of all the quantities$\left.\right]$

By integration the moment of inertia can be expressed in the following way :

$$I = \int r^2 dm \qquad \ldots \qquad \ldots \qquad \ldots \qquad (4.33)$$

where dm is the mass of infinitesimally small element of the body and r is the distance of that small part from rotational axis.

Moment of inertia does not depend on angular velocity of particles, but depends on distribution of particles about the axis of rotation.

Unit and dimension of moment of inertia :

In M.K.S. and S.I. system unit of moment of inertia is kilogram-metre (kg-m^2) and dimension is [I] = [mass × (distance)^2] = [ML^2]

4.21.2 Radius of gyration

Definition : If the total mass of a rigid body is assumed to be concentrated at a point and if the moment of inertia of that point mass with respect to a rotational axis is equal to the moment of inertia of that whole body, then the distance of that point from the axis is called the radius of gyration. It is denoted by K.

Explanation : Let B be a rigid body which is rotating with respect to an axis XY. The rigid body is composed of innumerable particles of masses $m_1, m_2, m_3 \ldots\ldots m_n$ and the particles are respectively at distances $r_1, r_2, r_3 \ldots\ldots r_n$ from the axis of rotation.

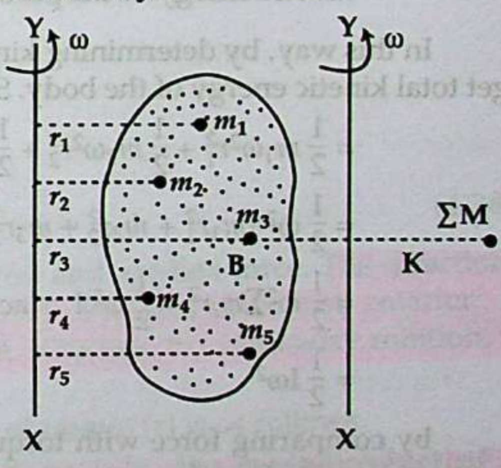


Fig. 4.26(a)

Now, suppose there is a point mass M at a distance of K from the rotational axis [Fig. 4.26(a)].

$$M = \sum m_i = (m_1 + m_2 + m_3 + \ldots\ldots + m_n)$$

clearly, moment of inertia in both cases will be same.

That means,

$$MK^2 = \sum m_ir_i^2 = (m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots\ldots + m_nr_n^2) \quad \ldots \quad (4.34)$$

$$\therefore\ K = \sqrt{\frac{m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots\ldots + m_nr_n^2}{M}}$$

$$= \sqrt{\frac{I}{M}} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (4.35)$$

Radius of gyration of a body with respect to a fixed axis is 0·2 m means that if moment of inertia is determined considering that total mass of the body is concentrated at a distance 0·2 m from that axis, then total moment of inertia is found out.

Example : Moment of inertia of solid sphere with respect to diameter is, $I = \frac{2}{3} MR^2$.

So, radius of gyration with respect to the diameter is,

$$K = \sqrt{\frac{I}{M}} = \sqrt{\frac{\frac{2}{5}MR^2}{M}} = \sqrt{\frac{2}{5}}\ R.$$

4·22 Rotational kinetic energy

Suppose a rigid body B rotating in a circular orbit with uniform angular velocity of ω around XY axis [Fig. 4·26]. For this rotation the body contains some kinetic energy. **This energy is called rotational kinetic energy.**

Let, linear velocity of particle m_1 be v_1, so $v_1 = \omega r_1$
linear velocity of particle m_2 be v_2, so $v_2 = \omega r_2$
linear velocity of particle m_3 be v_3, so $v_3 = \omega r_3$

So, kinetic energy of the particle $m_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 \omega^2 r_1^2$

kinetic energy of the particle $m_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 \omega^2 r_2^2$

kinetic energy of the particle $m_3 = \frac{1}{2} m_3 v_3^2 = \frac{1}{2} m_3 \omega^2 r_3^2$

In this way, by determining kinetic energies of all the particles and adding them we get total kinetic energy of the body. So, rotational kinetic energy of the body,

$$= \frac{1}{2} m_1 \omega^2 r_1^2 + \frac{1}{2} m_2 \omega^2 r_2^2 + \frac{1}{2} m_3 \omega^2 r_3^2 + \ldots\ldots\ldots +$$

$$= \frac{1}{2} \omega^2 [m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots]$$

$$= \frac{1}{2} \omega^2 \Sigma m_i r_i^2 = \frac{1}{2} \omega^2 . I \quad \text{[according to equation 4·34]}$$

$$= \frac{1}{2} I\omega^2 \qquad \ldots \qquad \ldots \qquad (4·36)$$

by comparing force with torque we find that the role played by a mass in linear motion, inertia plays the same role in rotational motion.

Now if $\omega = 1$, i.e., moment of inertia for a rotating body with unit angular velocity, $I = 2E$ or twice the kinetic energy. So, it can be said that **moment of inertia of a body rotating with unit angular velocity is two times the kinetic energy.**

4·23 Torque or Moment of a force

A rigid body can rotate about a point. For example, a photograph can rotate about the point of contact between the nail and the thread from which the photograph is hanging and rotating; besides, the wheel of a vehicle can rotate about its axis.

The torque or moment of force about a chosen axis is the product of the force and its moment arm. It is denoted by τ (Tau).

Explanation : Let us consider a thin sheet AB fixed in horizontal position in such a way as it can rotate about point O and about the perpendicular axis XOY [Fig. 4'27]. If the sheet is rotated by applying a force on any point, say C, it is observed that,

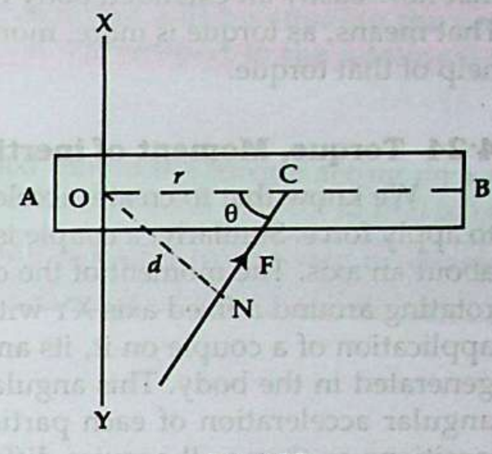


Fig. 4'27

1. Greater the magnitude of applied force, greater is the power to rotate.

2. Greater the distance d between point O and the point of application of force F, the greater is the power to rotate.

3. If the line of action of the force is parallel to the plane of the sheet, it will not rotate.

Because of the above reasons, **magnitude of the moment of force or torque is measured by the product of the magnitude of force and the perpendicular distance *d* of the action of force from the axis of rotation.**

$\therefore \quad \tau = d \times F$ ... ... (4'37)

or, **Torque or Moment of Force = Force × Perpendicular distance**

In fig. 4'27 from O distance of the point of action of the force $\vec{F} = r$, the line of action NC of the force = d and $\angle NCO = \theta$.

So, $ON = d = r \sin\theta$

$\therefore \; \tau = d \times F = r F \sin\theta$

In vector form τ can be expressed as,

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (4'38)$$

Here $\vec{r}$ and $\vec{F}$ are respectively position vector and applied force. The direction of τ is perpendicular to the plane containing r and F. For anti-clockwise rotation, the direction of τ is upward and its value is positive; whereas, for clockwise rotation, τ is downward and its value is negative.

According to equation (4'38), the definition of torque (τ) is as follows.

Definition : For an object moving about an axis, the cross-product of the position vector of a point on the object where the force is acting and the applied force is called torque.

Unit of torque or moment of a force : S. I. unit of τ is Newton-metre (N-m).

Dimension of torque or moment of force

From the definition of torque or moment of force its dimension can be derived. Dimensional equation of moment of force.

[torque or moment of force] = [force × distance] = $[MLT^{-2} \times L] = [ML^2T^{-2}]$

Significance of torque : From the torque with respect to an axis it is understood that how easily an extended body of fixed mass can be rotated with respect to that axis. That means, as torque is more, more easily the angular velocity can be changed with the help of that torque.

4·24 Torque, Moment of inertia and Angular accelaration

We know that to create acceleration of an object in linear uniform motion we need to apply force. Similarly, a couple is needed to generate acceleration on an object moving about an axis. The moment of the couple so applied is called torque. Suppose a body is rotating around a fixed axis XY with uniform angular velocity ω [Fig. 4·20]. Now due to application of a couple on it, its angular velocity will increase, i.e., acceleration will be generated in the body. This angular acceleration generated in the body is equal to the angular acceleration of each particle in it. But since particles are located at different positions so they will acquire different linear accelerations. Larger the distance of the particles from the axis, larger will be the linear acclerations.

Let the object be composed of particles of masses m_1, m_2, m_3 etc. and respective distances of the particles from the axis of rotation be respectively r_1, r_2, r_3 etc.

Now, according to the above discussion, the angular acceleration of the object or its particles is, $\alpha = \frac{d\omega}{dt}$ where ω is the angular velocity.

Now, linear acceleration of the particle of mass m_1 at a distance r_1 from the axis $= r_1\frac{d\omega}{dt}$

∴ Force applied on that particle $= m_1 r_1 \frac{d\omega}{dt}$

∴ Moment of the force or torque on the particle with respect to the axis of rotation τ = force × distance of the particle from the axis of rotation

$$= m_1 r_1 \frac{d\omega}{dt} \times r_1 = m_1 r_1^2 \frac{d\omega}{dt}$$

Similarly, for particles of masses m_2, m_3, m_4 etc., moments of the force are respectively, $m_2 r_2^2 \frac{d\omega}{dt}$, $m_3 r_3^2 \frac{d\omega}{dt}$, $m_4 r_4^2 \frac{d\omega}{dt}$ etc.

Then the summation of the above moments is the torque applied on the object.

$$\tau = m_1 r_1^2 \frac{d\omega}{dt} + m_2 r_2^2 \frac{d\omega}{dt} + m_3 r_3^2 \frac{d\omega}{dt} + \cdots \cdots \cdots$$

$$= (m_1 r_1^2 + m_2 r_2^2 + r_3 r_3^2 + \ldots) \frac{d\omega}{dt} = (\Sigma m r^2) \frac{d\omega}{dt}$$

$$\therefore \tau = I \frac{d\omega}{dt} = I\alpha \quad [\because \Sigma m r^2 = I]$$

$$\therefore \tau = I \frac{d\omega}{dt} = I\alpha \quad \ldots \quad \ldots \quad \ldots \quad (4·39)$$

where I is the moment of inertia and α, the angular acceleration.

or, torque = moment of inertia × angular acceleration. Torque of the couple acting on the particle rotating with angular acceleration will be equal to the product of the moment of inertia and angular acceleration with respect to the rotational axis.

Again, if $\frac{d\omega}{dt} = 1$, then $\tau = I$.

∴ Unit angular acceleration that is producd due to the torque acting on a rigid body rotating around an axis is called moment of inertia with respect to that axis.

Verify : Show that the torque acting on a body is equal to the rate of change of angular momentum.

We know,

angular momentum, $\vec{L} = \vec{r} \times \vec{p}$

or, $\vec{L} = \vec{r} \times m\vec{v}$

$$\therefore \quad \frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt} = \vec{v} \times m\vec{v} + \vec{r} \times m\vec{a}$$

$$= \vec{v} \times m\vec{v} + \vec{r} \times \vec{F} = \vec{r} \times \vec{F} \quad [\because \vec{v} \times \vec{v} = 0]$$

So, the torque acting on a body is equal to the rate of change of angular momentum.

Mathematical examples

1. Mass of a wheel is 5 kg and radius of gyration about an axis is 0·2m. What is its moment of inertia ? In order to produce angular acceleration of 2 rad s^{-2} in the wheel what magnitude of torque is to be applied ?

We know,

$I = MK^2$
$= 5 \times (0{\cdot}2)^2$
$= 5 \times 0{\cdot}04$
$\therefore I = 0{\cdot}2$ kg m^2

Here,
$M = 5$ kg
$K = 0{\cdot}2$ m
$I = ?$

Again,

$\tau = I\alpha$
$= 0{\cdot}2 \times 2 = 0{\cdot}4$
$\therefore \tau = 0{\cdot}4$ N-m

Here,
$I = 0{\cdot}2$ kgm^2
$\alpha = 2$ rad s^{-2}
$\tau = ?$

2. Mass of a metallic sphere is 6 g. It is fastened at one end of a thread of length of 3m and is rotated 4 times per second. What is its angular momentum ?

We know,

$L = I\omega$

$= mr^2 \frac{2\pi N}{T}$

$= \frac{0{\cdot}006 \times (3)^2 \times 2 \times 3{\cdot}14 \times 4}{1}$

$= 1{\cdot}366$ kgm^2s^{-1}

Here,
Mass of the sphere, $m = 6$ g $= 0{\cdot}006$ kg
Length of the thread or radius of the circular path, $r = 3$ m
No. of rotation per sec, $N = 4$ times
time, $t = 1$ sec
Angular momentum, $L = ?$

3. Mass of a wheel is 5 kg and radius of gyration is 25 cm. What is its moment of intertia ? What magnitude of torque is to be applied in order to create 4 rad s^{-2} angular acceleration of the wheel ?

We know,

$$I = MK^2$$
$$\therefore I = 5 \times (0·25)^2 = 0·3125 \text{ kg}^2\text{m}^2$$

Here,
$M = 5$ kg
$K = 25$ cm $= 0·25$ m
$I = ?$

Again, $\tau = I\alpha$

$$\therefore \tau = 0·3125 \times 4 = 1·25 \text{ N-m}$$

Here,
$I = 0·3125$ kg-m^2
$\alpha = 4$ rads^{-2}
$\tau = ?$

4. A boy weighing 40 kg sitting in a merry-go-round is rotated in a circular orbit of 20 m diameter with angular velocity of 6 rpm. Calculate the angular momentum of the boy.

We know,

$$L = I\omega = mr^2\omega = 40 \times (10)^2 \times \frac{1}{5}\pi \text{ kg m}^2\text{s}^{-1} = 2·512 \times 10^3 \text{ kg m}^2\text{s}^{-1}$$

Here,
$\omega = \frac{6 \times 2\pi}{60} = \frac{1}{5}\pi$ rad s^{-1}
$m = 40$ kg
$r = \frac{d}{2} = \frac{20 \text{ m}}{2} = 10$ m

5. Planet mars is rotating in a circular orbit of radius of $2·28 \times 10^{11}$ m around the sun. Calculate its angular momentum. Mass of mars is $6·46 \times 10^{23}$ kg and its time period is $5·94 \times 10^7$ s.

We know, angular momentum

$$L = I\omega = mr^2 \times \omega = mr^2 \times \frac{2\pi}{T} = \frac{6·46 \times 10^{23} \times (2·28 \times 10^{11}) \times 2 \times 3·14}{5·94 \times 10^7} = 3·55 \times 10^{39} \text{ kgm}^2\text{s}^{-1}$$

Here,
radius, $r = 2·28 \times 10^{11}$ m
mass, $m = 6·46 \times 10^{23}$ kg
time period, $T = 5·94 \times 10^7$ s
angular momentum, $L = ?$

6. If radius vector $\vec{r} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and force vector, $\vec{F} = 2\hat{i} + 2\hat{j} + 2\hat{k}$, then calculate the torque τ.

$$\text{torque, } \vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ 2 & 2 & 2 \end{vmatrix} = \hat{i}(6-4) - \hat{j}(4-4) + \hat{k}(4-6) = 2\hat{i} - 0 - 2\hat{k} = 2\hat{i} - 2\hat{k}$$

7. Mass of a rotating body is 2 kg. Its distance from the rotation axis is 1 m. If the body is rotated with angular velocity of 5 rad s^{-1}, then what will be the kinetic energy of the body ?

We know,

$$E_k = \frac{1}{2}I\omega^2 = \frac{1}{2} \times mr^2 \times \omega^2 = \frac{1}{2} \times 2 \times 1^2 \times (5)^2 = 25 \text{ J}$$

4·25 Two theorems relating moment of inertia

There are two simple theorems for determination of moment of inertia of a rigid body with respect to a particular axis.

One of the two theorems is called (1) perpendiculer axes theorem and the other one is called (2) parallel axes theorem.

4·25·1 Perpendicular axes theorem

The sum of two moments of inertia of a plane lamina with respect to two mutually perpendicular axes in the plane of the lamina is equal to the moment of inertia with respect to the perpendicular axis drawn at the point of intersection of the two axes in that lamina.

Explanation : Let the moments of inertia along two mutually perpendicular axes OX and OY on a plane lamina be respectively I_x and I_y. Suppose the moment of inertia along the perpendicular OZ drawn at the point of intersection of the two axes on that lamina be I_z. It is to be proved that $I_x + I_y = I_z$

Drawing : Let us take a plane lamina. Let us draw two mutually perpendicular axes OX and OY on this lamina [Fig. 4·28].

Now let us draw a normal at the point of insersection O of the two axes OX and OY.

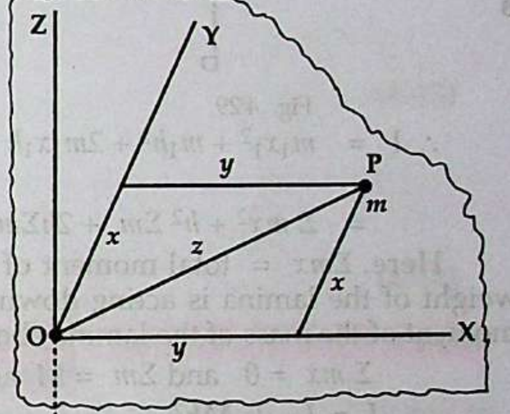


Fig. 4·28

Proof : Let us take a point P on the plane lamina whose co-ordinates are x, y and z. Now consider a particle of mass m at point P. Moment of inertia of the particle with respect to the axis OZ $= mz^2$.

$\therefore$ moment of inertia of the whole lamina with respect to OZ,

$$I_z = \Sigma\, mz^2 = \Sigma\, m\,(x^2 + y^2)$$

$$= \Sigma mx^2 + \Sigma my^2 \quad \cdots \quad \cdots \quad \cdots \quad (4.40)$$

but $\Sigma my^2 = I_x$ and $\Sigma\, mx^2 = I_y$

So, from equation (4.40) we get,

$$I_z = I_y + I_x$$

$$\text{or, } I_z = I_x + I_y \quad \cdots \quad \cdots \quad \cdots \quad (4.41)$$

$\therefore$ the theorem is proved.

4·25·2 Parallel axes theorem

The moment of inertia of a lamina about any axis is equal to the sum of the moment of inertia of the lamina about a parallel axis passing through the centre of mass and the product of the mass of the lamina and the square of perpendicular distance between the two axes.

Explanation : Let AB be any axis on the plane of the paper and CD is another axis parallel to it. The axis CD is passed through the centre of mass G of the plane lamina of mass M [Fig. 4·29]. If the distance between parallel axes AB and CD is h, and if the moments of inertia of the lamina with respect to AB and CD are respectively I and I_G, then according to the theorem it is to be proved that, $I = I_G + Mh^2$

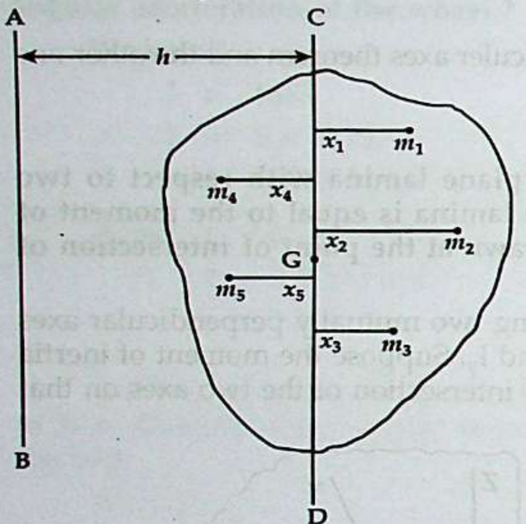


Fig. 4·29

Proof : Let the lamina is composed of particle of masses m_1, m_2, m_3 etc. Distances from the axis CD of the particles are respectively x_1, x_2, x_3 etc, then moment of inertia of the particle of mass m_1 with respect to the axis AB $= m_1(x_1 + h)^2 = m_1x_1^2 + m_1h^2 + 2m_1x_1h$

Similarly, moment of inertia of the particle of mass m_2 with respect to the axis AB $= m_2x_2^2 + m_2h^2 + 2m_2x_2h$; moment of inertia of the particle of mass m_3

$$= m_3x_3^2 + m_3h^2 + 2m_3x_3h \text{ etc.}$$

∴ If I is moment of inertia of the whole lamina with respect to the axis AB, then it will be equal to the summation of the above moments of inertia.

$$\therefore I = m_1x_1^2 + m_1h^2 + 2m_1x_1h + m_2x_2^2 + m_2h^2 + 2m_2x_2h + m_3x_3^2 + m_3h^2 + 2m_3x_3h + \dots \dots$$

$$= \Sigma mx^2 + h^2\Sigma m + 2h\Sigma mx.$$

Here, Σmx = total moment of the mass of the whole lamina about CD. But the weight of the lamina is acting downward along the line CD through the point G, so moment of the mass of the lamina about the axis CD,

$$\Sigma mx = 0 \text{ and } \Sigma m = M \text{ and } I_G = \Sigma mx^2$$

$$\therefore I = I_G + Mh^2 \quad \dots \quad \dots \quad \dots \quad (4.42)$$

4·26 Determination of moment of inertia and radius of gyration for some special cases

(1) Moment of inertia and radius of gyration of a thin uniform rod about an axis through its centre and perpendicular to its length

Let AB be a thin uniform rod of length l and mass M, free to rotate about the axis CD which is passing through the centre O and perpendicular to the length of the rod [Fig. 4·30]. The moment of inertia about the axis CD and radius of gyration are to be found out.

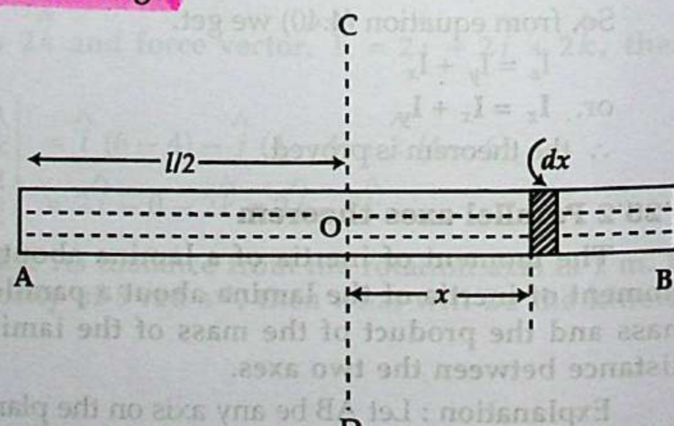


Fig. 4·30

Since the rod is uniform, the mass per unit length $= \frac{M}{l}$ · So, at a distance x from the axis CD let dx

be a small length whose mass is dM, then $dM = \frac{M}{l}dx$. As dx is very small, we can consider that all the particles in dx are at a distance x from CD. So, moment of inertia of dx about axis CD $= dM \times x^2 = \frac{M}{l} \times dx \times x^2$.

Now integrating the above equation within limits $x = l/2$ and $x = -l/2$ we get moment of inertia for the entire rod.

∴ Moment of inertia of the rod about the axis CD is,

$$I = \int_{-l/2}^{l/2} \left(\frac{M}{l}\right) x^2 dx = \frac{M}{l} \int_{-l/2}^{l/2} x^2 dx$$

$$= \frac{M}{l}\left[\frac{x^3}{3}\right]_{-l/2}^{l/2} = \frac{M}{l}\left[\frac{l^3}{3 \times 8} + \frac{l^3}{3 \times 8}\right]$$

$$= \frac{M}{l} \times \frac{2l^3}{24} = \frac{Ml^2}{12}$$

$$\therefore \quad I = \frac{M}{12} l^2 \qquad \dots \qquad \dots \qquad \dots \qquad (4.43)$$

Let K be the radius of gyration

$$\therefore \quad MK^2 = I = \frac{M}{12} l^2$$

$$\text{or,} \quad K = \frac{l}{\sqrt{12}} = \frac{l}{2\sqrt{3}} \qquad \dots \qquad \dots \qquad \dots \qquad (4.44)$$

2. Moment of inertia and radius of gyration of a thin uniform rod about an axis passing through one end and perpendicular to its length

Let us consider a thin and uniform rod. Its mass is M and length is l. The rod is rotating around CD, one of its end A and perpendicular to its length [Fig. 4·31]. The moment of inertia and the radius of gyration of the rod around this CD are to be determined.

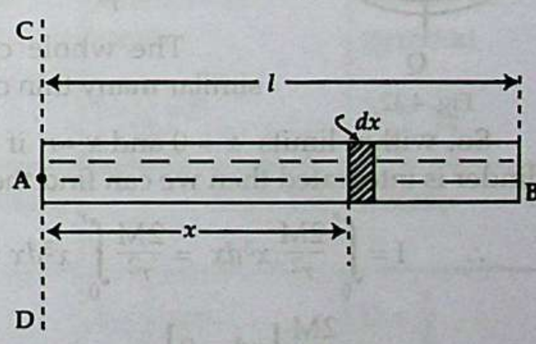


Fig. 4·31

According to the description, as the rod is uniform its mass per unit length is $\frac{M}{l}$.

So, mass of a small element of length dx, $dM = \frac{M}{l} \times dx$. As the element is very small, so each of its particles can be considered to be at a distance of r from the axis CD.

Now, if it is integrated within limits $x = 0$ and $x = l$, then moment of inertia of the whole rod with respect to CD will be found out

$$\therefore \text{ required moment of inertia, } I = \int_{x=0}^{x=l} \left(\frac{M}{l}\right) \times dx \times x^2 = \frac{M}{l} \int_{x=0}^{x=l} x^2 dx$$

$$= \frac{M}{l}\left[\frac{x^3}{3}\right]_0^l = \frac{M}{3l} \times l^3$$

$\therefore\ I = \frac{1}{3} Ml^2$ ... ... (4.45)

Now, if K is the radius of gyration, then $MK^2 = \frac{1}{3}Ml^2$

$\therefore\ K = \frac{l}{\sqrt{3}}$ ... ... (4.46)

3. Moment of inertia and radius of gyration of a solid cylinder rotating about its own axis

Let mass of a uniform solid cylinder C be M, length l and radius r [Fig. 4·32]. It is rotating around its axis PQ. Moment of inertia and radius of gyration with respect to PQ are to be determined. According to the description, volume of the cylinder $= \pi r^2 \times l$

density of the material of the cylinder $= \frac{\text{mass}}{\text{volume}} = \frac{M}{\pi r^2 l}$

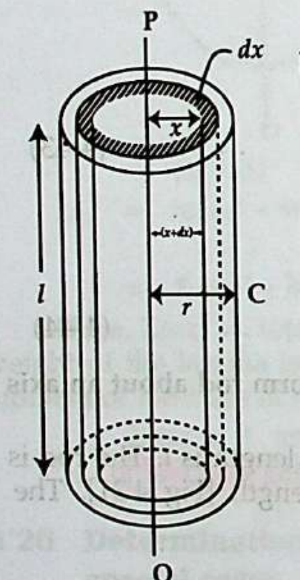


Fig. 4·32

Let a hollow coaxial thin cylinder of radius r and width dx be considered

Area of this thin cylinder $= 2\pi x dx$

volume $= 2\pi x \times dx \times l$ and mass = volume × density

$= 2\pi x \times dx \times l \times \frac{M}{\pi r^2 l}$

$= \frac{2Mxdx}{r^2}$

Since the cylinder of width dx is very thin hence its each particle can be considered to be at equal distance x from PQ. So, moment of inertia of this thin cylinder with respect to PQ

$= \frac{2Mxdx}{r^2} \times x^2 = \frac{2M}{r^2} x^3 dx$

The whole cylinder may be considered to be formed by similar many thin cylinders.

So, within limits $x = 0$ and $x = r$ if the moment of inertia of the above hollow thin cylinder is integrated then we can find the moment of inertia I of the whole cylinder.

$$\therefore\quad I = \int_0^r \frac{2M}{r^2} x^3 dx = \frac{2M}{r^2}\int_0^r x^3 dx = \frac{2M}{r^2}\left[\frac{x^4}{4}\right]_0^r$$

$$= \frac{2M}{4r^2}\left[r^4 - 0\right]$$

$$\therefore\quad I = \frac{1}{2} Mr^2 \quad \ldots \ldots \ldots \quad (4.47)$$

In this case if radius of gyration is K, then

$MK^2 = \frac{1}{2}Mr^2$

$$\therefore\quad K = \frac{r}{\sqrt{2}} \quad \ldots \ldots \ldots \quad (4.48)$$

Work : What are the moment of inertia and radius of gyration (i) through the mid point of a thin and uniform bar of mass M and length l, (ii) with respect to the axis passing through one end, (iii) through normal passing through the centre of a thin disc of mass M and radius r and (iv) with respect to the axis of a solid cylinder of mass M and radius r.

(i) Moment of inertia for thin and uniform bar rotating through the mid point is $\frac{Ml^2}{12}$, radius of gyration is $\frac{l}{\sqrt{12}}$.

(ii) Moment of inertia of a thin and uniform bar for rotating around the axis passing through one end of the bar is $\frac{Ml^2}{3}$.

(iii) Moment of inertia and radius of gyration of a thin disc of mass M and radius r rotating through normal of the disc are respectively $\frac{1}{2}Mr^2$ and $\frac{r}{\sqrt{2}}$.

(iv) Moment of inertia and radius of gyration of a solid cylinder of mass M and radius r rotating around the axis of the cylinder are respectively $\frac{1}{2}Mr^2$ and $\frac{r}{\sqrt{2}}$.

4·27 Equations of moment of inertia and radius of gyration with respect to location of rotational axes

Matter	Moment of inertia and radius of gyration according to the location of rotational axis			
	With respect to the normal axis passing through the centre		With respect to the normal axis passing through the end point	
Straight bar (length = l)	Moment of inertia $I = \frac{l}{12}ml^2$	Radius of gyration $K = \frac{l}{2\sqrt{3}}$	Moment of inertia $I = \frac{1}{3}ml^2$	Radius of gyration $K = \frac{l}{\sqrt{3}}$
	w.r.t. to the normal axis passing through centre		w.r.t. any diameter	
Circular disc (radius = r)	$I = \frac{1}{2}mr^2$	$K = \frac{r}{\sqrt{2}}$	$I = \frac{1}{4}mr^2$	$K = \frac{r}{2}$
	w.r.t. to the normal axis passing through centre		w.r.t. any diameter	
Cylindrical disc (internal radius = r external radius = R)	$I = \frac{1}{2}m(R^2 + r^2)$	$K = \sqrt{\frac{R^2 + r^2}{2}}$	$I = \frac{1}{4}(R^2 + r^2)$	$K = \frac{(R^2 + r^2)^{\frac{1}{2}}}{2}$

	w.r.t. to the normal axis passing through centre		w.r.t. any axis passing through the centre and parallel to the width	
Rectangular plate (length = l, width = b)	$I = \frac{1}{12}(l^2 + b^2)$	$K = \frac{1}{2}\sqrt{\frac{l^2 + b^2}{3}}$	$I = \frac{1}{12} ml^2$	$K = \frac{l}{2\sqrt{3}}$
	w.r.t. the axis of the cylinder		w.r.t. any axis normal to the centre of the length	
Solid cylinder (length = l, radius = r)	$I = \frac{1}{2} mr^2$	$K = \frac{r}{\sqrt{2}}$	$I = m\left(\frac{r^2}{4} + \frac{l^2}{12}\right)$	$K = \sqrt{\frac{r^2}{4} + \frac{l^2}{12}}$
	w.r.t. to the axis normal to the centre		w.r.t. any diameter	
Circular ring (radius = r)	$I = mr^2$	$K = r$	$I = \frac{1}{2} mr^2$	$K = \frac{r}{\sqrt{2}}$
	w.r.t. any diameter		w.r.t. any tangent	
Thin spherical shell (radiuds = r)	$I = \frac{2}{3} mr^2$	$K = \sqrt{\frac{2}{3}}\, r$	$I = \frac{5}{3} mr^2$	$K = \sqrt{\frac{5}{3}}\, r$

4·28 Experimental

Name of the experiment : Piriod : 2	Determination of moment of inertia of a fly wheel

Theory : Suppose, angular velocity of a wheel is ω and its radius is r. Then linear velocity of the wheel is, $v = \omega r$. If the moment of inertia of a body is I and the wheel is rotating around an axle, then its

rotational kinetic energy, $E = \frac{1}{2} I\omega^2$ ... ... (i)

Work done against friction for each complete rotation of the wheel is W. If the total number of complete rotation of the wheel is n_1 before the body of mass m falls on the ground, then total energy = Wn_1. If the body of mass m falls from a height h, then,

Potential energy $= mgh$ ... ... (ii)

We know, potential energy of the axle = kinetic energy of the axle + rotational kinetic energy of the wheel + total work of the wheel

or, $mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 + Wn_1$

or, $mgh = \frac{1}{2} m\, \omega^2 r^2 + \frac{1}{2} I\omega^2 + Wn_1$ ... ... (iii)

The rotating wheel stops completing n_2 number of rotation after the body of mass m, attached at the end of the axle by a thread, has been separated from the axle, so work done against the friction = Wn_2

$$\therefore \quad Wn_2 = \frac{1}{2} I\omega^2$$

$$\text{or,} \quad W = \frac{I\omega^2}{2n_2} \quad \ldots \quad \ldots \quad \ldots \quad \text{(iv)}$$

In equation (iii), the value of W is inserted and we get,

$$mgh = \frac{1}{2} m\omega^2 r^2 + \frac{1}{2} I\omega^2 + \frac{1}{2}\left(I\frac{\omega^2}{n_2}\right) \times n_1$$

$$\text{or, } mgh = \frac{1}{2} m\omega^2 r^2 + \frac{1}{2} I\omega^2\left(1 + \frac{n_1}{n_2}\right)$$

$$\text{or, } 2mgh = m\omega^2 r^2 + I\omega^2\left(1 + \frac{n_1}{n_2}\right)$$

$$\text{or, } I\omega^2\left(1 + \frac{n_1}{n_2}\right) = 2mgh - m\omega^2 r^2$$

$$\text{or, } I = \frac{2mgh - m\omega^2 r^2}{\omega^2\left(1 + \frac{n_1}{n_2}\right)} \quad \ldots \quad \ldots \quad \ldots \quad \text{(v)}$$

Apparatus : An iron axle, a heavy wheel, some ropes, a mass, stop watch, metre scale, slide callipers.

Description of the apparatus : An axial rod B, attached to a heavy rotating wheel, on which one end of a wrapped thread is tied and a mass m is attached to the other end of the thread by which the wheel can be rotated.

Procedure : (1) First of all, let us measure the radius of the axle by a slide callipers.

(2) Then for the determination of number of rotation a mark by a chalk is put on the axle and a rope is wound on the axle. At the other end of the rope a mass m is fastened and if it is dropped from position R, the wheel after rotating a few times, the weight with the rope will fall to position S. The wheel makes n_1 number of rotation to touch the point S and time for this drop is noted from the stop watch.

Now the rope is again wound on the axle and the mass is fastened on the other end of the rope. From position R the mass is allowed to fall to the ground and as soon as it touches the ground, the stop watch is started. When the axle comes to rest the stop watch is stopped. Total time and number of rotations of the wheel before it comes to rest are noted i.e., total number of rotation (n_2) is noted.

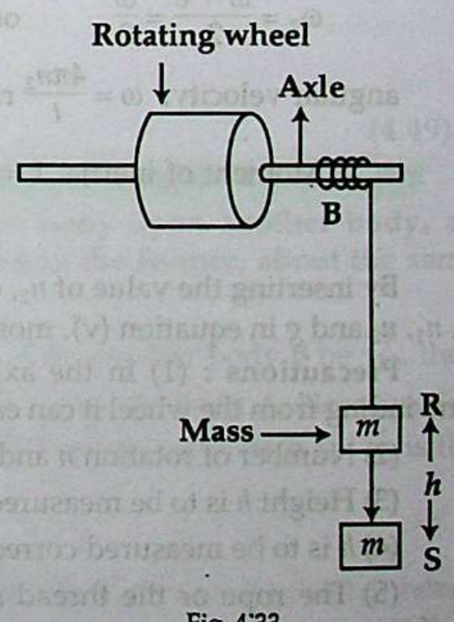


Fig. 4·33

Table-1 : Radius (r) of the axle B

No. of observation	Main scale reading M (cm)	Vernier scale reading n	Vernier constant c (cm)	Vernier scale reading cm $F = c \times n$	Total reading = (M + D) cm	Average diameter D (cm)	Radius $r = \frac{D}{2}$ (cm)
1							
2							
3							

Table-2 : Determination of time and number of rotation

No. of observation	Number of rotation of the wheel (n_2)	Time for rotations t sec	Total number of rotations n_2	Average time for average numbers of rotation t sec
1				
2				
3				

Calculation : If the axis takes time t for n_2 number of rotation, then average angular velocity, $\omega_2 = \frac{2\pi n_2}{t}$

The axle acquires zero velocity with uniform retardation from angular velocity ω, so its average angular velocity

$$\omega_2 = \frac{\omega + 0}{2} = \frac{\omega}{2} \quad \text{or,} \quad \frac{2\pi n_2}{t} = \frac{\omega}{2}$$

angular velocity, $\omega = \frac{4\pi n_2}{t}$ rad s^{-1}

$$\therefore \text{ Moment of inertia, } I = \frac{2mgh - m\omega^2 r^2}{\omega^2\left(1 + \frac{n_1}{n_2}\right)} = \ldots\ldots \text{ g-cm}^2 = \ldots\ldots\ldots\ldots \text{ kg-m}^2$$

By inserting the value of n_2, ω can be found out. By inserting the values of m, ω, r, h, n_1, n_2 and g in equation (v), moment of inertia of the heavy wheel can be found out.

Precautions : (1) In the axle rope is to be wounded in such a way that while unwinding from the wheel it can easily drop on the ground.

(2) Number of rotation n and time t is to be measured correctly.

(3) Height h is to be measured from the mark on the axle.

(4) h is to be measured correctly.

(5) The rope or the thread should be light and wrapping on the rod should be uniform.

4·29 Newton's laws for angular motion

Newton's laws for rotatory motion are very similar to those for linear motion. Corresponding to three laws in the case of linear motion, there are also three laws of rotational motion. These laws are discussed below.

(1) First Law : The first law applies to a condition of equilibrium. The law is stated as follows :

A body does not change its angular velocity unless it is acted upon by an external, unbalanced torque.

Explanation : A body at rest does not begin to rotate without a torque to cause it to do so. Neither does a body that is rotating change its motion nor change its axis unless a torque acts.

Example : A rotating wheel would continue to rotate for ever if not stopped by a torque such as that due to friction.

(2) Second law : **The rate of change of angular momentum of a body rotating about a fixed axis in it, is directly proportional to the external torque applied and takes place in the direction of the torque.**

Explanation : We know, angular momentum, L = Iω. Now according to this law, rate of change of L i.e., $\frac{dL}{dt}$ is proportional to the torque applied i.e.,

$$\tau \propto \frac{dL}{dt} \propto I\frac{d\omega}{dt}$$

$$\propto I\alpha \quad [\because \alpha = \frac{d\omega}{dt}]$$

$$\text{or,} \quad \tau = KI\alpha$$

Here K is a porportionality constant.

In S. I. unit K = 1

$$\therefore \quad \vec{\tau} = I\vec{\alpha} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (4.49)$$

Rate of change of angular momentum dL will take place along the direction of τ.

(3) Third law : **If a torque be applied by one body upon another body, an equal and opposite torque is applied by the latter upon the former, about the same axis of rotation.**

Explanation : Let the torque applied by a body A on another body B be $\vec{\tau}_{12}$, then the body B will also exert an equal and opposite torque $\vec{\tau}_{21}$ on the body A. Here applied torque on B by A is $\vec{\tau}_{12}$ is the action torque and applied torque on A by B is $\vec{\tau}_{21}$ is the reaction torque.

$$\therefore \quad \vec{\tau}_{12} = -\vec{\tau}_{21} \text{ and } \tau_{12} = \tau_{21}$$

The direction of the reaction torque τ_{21} is opposite to the action torque τ_{12}, hence negative sign is used.

4·30 Centripetal and centrifugal force

4·30·1 Centripetal force

According to Newton's first law due to inertia of motion a moving body always intends to move in a straight line with uniform speed. So unless an external force is applied motion of a body does not change automatically. In every moment direction of motion of a rotating body in a circular path changes; so a force always acts on this body from outside.

We have seen before that when a body of mass m rotates in a circular path of radius r with speed v, then a centripetal acceleration $\frac{v^2}{r}$ acts on that body. According to Newton's second law of motion this acceleration is produced due to the action of a force. Clearly, this force will be centripetal as well i.e., will act along the radius towards the centre and its magnitude will be equal to the product of the mass of the body and the centripetal acceleration i.e., $\frac{mv^2}{r}$ · If the action of this force stops for any reason, then there will not be any force left to rotate the body in the circular path. Then the body will go away along the tangent of the circle and will move with uniform speed in a straight line.

The action of force for which a body moves in a circular path with uniform speed and the force which acts perpendicularly to the direction of motion i.e., towards the centre of the circle is called centripetal force.

If a body of mass m rotates around a circular path of radius r with uniform speed v, then the magnitude of the centripetal force is $\frac{mv^2}{r}$ · In angular velocity it is expressed as $m\omega^2 r$.

Centripetal force is a no-work force

Centripetal force always acts perpendicular to the direction of motion, hence there is no displacement along the direction of the force. So, the centripetal force does not do any work. For this reason centripetal force is called **no-work force.**

Centrifugal reaction : Centripetal force acting on the body rotating in a circular path is applied from outside. The body which applies this outside force, according to Newton's third law of motion equal and opposite force is applied on that body by the first body. Obviously, this reaction force acts along the radius vector towards outside. This is called **centrifugal reaction.**

Suppose by fastening a piece of stone with a thread is rotated in a circular path [Fig. 4·34]. All the time a centripetal force F_C is acting on the stone by the thread. Here tension of the thread is the necessary centripetal force. If the thread is torn off, then action of the centripetal force stops; instantaneously the stone swiftly moves away along the tangent of the circle with uniform speed in a straight line. While rotating the stone by the hand in a circular path, the stone also applies equal and opposite reaction F_R on the hand; as a result, the hand feels a tension outward. Like other action-reaction here also F_C and F_R do not act on the same body, but act on two different bodies, for example, respectively on the stone and on the hand. When the thread tears off then two forces disappear simultaneously.

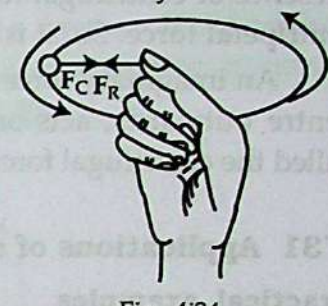


Fig. 4·34

Many other examples may be cited on centripetal force. Each planet in the universe revolves round the sun. The gravitational attraction force that the sun applies on those planets, acts as centripetal force on the planets.

Do yourself : Centripetal force of a body rotating in a circular path changes with the change of radius—explain.

4·30·2 Centrifugal force

We have seen earlier that for every body moving in a circular path a force acts towards the centre of the circle i.e., a centripetal force acts. The earth rotates around the sun. Here gravitational attraction force of the sun on the earth is the centripetal force. Naturally question may arise— by which force the centripetal force is balanced ? For what hindrance the earth does not go straight to the sun ? Apparently, it appears that another equal and opposite force acts on the earth. **This apparent force is called centrifugal force.** Obviously, centrifugal force is equal and opposite to the centripetal force. But it should be remembered that there is no real existence of centrifugal force. So actually the centripetal force is not balanced by any real force. Centrifugal force is an imaginary force.

For every body rotating in a circular orbit has a tendency to move away along the tangent of the circle, for example, in case of rotation of a stone by a thread if the thread is torn off then the thread speeds away along the tangent of the circle. Suppose, a body moving in a circular path is at position A at any instant [Fig. 4·35]. If no centripetal force acts along the direction of the centre of the circle O, then the body would have reached to point B along the tangent. But since the centripetal force acts, so the body approaches slightly towards the centre and reaches to point C, a point in the circle, instead of point B. That means, due to the influence of centripetal force the body is forced to move at every moment along the circular path. So, the centripetal force is always active in rotational motion.

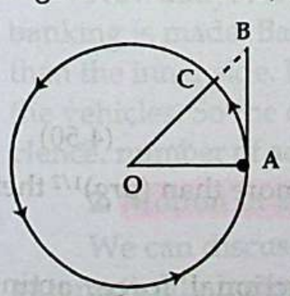


Fig. 4·35

Hence, the centripetal force is not balanced by any real force. For this reason, the presence of centrifugal force is apparently considered as real which acts opposite to the centripetal force. So, it is called **pseudo force.**

An imaginary force equal and opposite to the centripetal force i.e., opposite to the centre outwards, acts on a body rotating in a circular orbit with uniform speed. It is called the centrifugal force.

4·31 Applications of centripetal and centrifugal force

Practical examples

1. **Banking of roads :**

(a) Turning of a car/vehicle in a horizontal road : Suppose a vehicle is taking turn in a horizontal road. Here frictional force between the wheel of the vehicle and the road provides the necessary centripetal force for taking turn. This friction is static friction and self controlled. While taking turn the wheels of the vehicle tend to skid away. Frictional force acting towards the centre resists the skidding. If the vehicle takes turn moving very fast then the magnitude of the necessary centripetal force also increases enormously. But magnitude of the frictional force cannot be more than a certain limit. So if the vehicle takes turn with a great speed, then frictional force cannot provide necessary centripetal force. Hence the vehicle skids away from the road.

Suppose a vehicle of mass m is taking turn in a circular path of radius r with a velocity v. If the frictional force between the wheel and the road is F, then condition for safe turning of the vehicle will be,

$$F = \frac{mv^2}{r}$$

As the magnitude of v increases the vehicle takes turn with higher speed. But maximum value of F is μmg ; here μ is the co-efficient of static friction i.e., $F \leq \mu mg$.

So, condition of safe turning of the vehicle is

$$\frac{mv^2}{r} \leq \mu mg$$

$$\text{or,} \quad v^2 \leq \mu rg$$

$$\text{or,} \quad v \leq (\mu rg)^{1/2} \qquad \ldots \qquad \ldots \qquad (4.50)$$

If the speed of the vehicle is more than the above value, i.e., more than $(\mu rg)^{1/2}$ then the vehicle will skid away from the road.

(b) Turning of a vehicle in a road having banking : Frictional force, acting between the wheel of the vehicle and a horizontal road, damages the wheels. In order to decrease the energy and also to avoid accident due to skidding in every turn the outer part of the road is made slightly higher than the inner part. That is, the road slightly slopes at the centre of the banking. This is called **banking of a road**. Part of necessary centripetal force for taking turn of the vehicle comes from the horizontal component of the applied reaction force and the rest amount comes from the friction. If the angle of banking is correct, then from the horizontal component of the reaction necessary centripetal force is obtained; then frictional force has no role at all.

Here two torces are acting on the vehicle— (i) weight of the vehicle W acts directly downward and (ii) applied reaction R by the road acts vertically upward perpendicular to the plane of the road [Fig. 4·36]. Let the plane of the road be inclined with an angle θ with the horizontal plane; θ is called the **angle of banking**. Vertical component of reaction R is R cos θ balances the weight of the vehicle and the horizontal component R sin θ provides the necessary centripetal force. If the mass of the vehicle is m, its speed is v and radius of the banking of the road is R, then

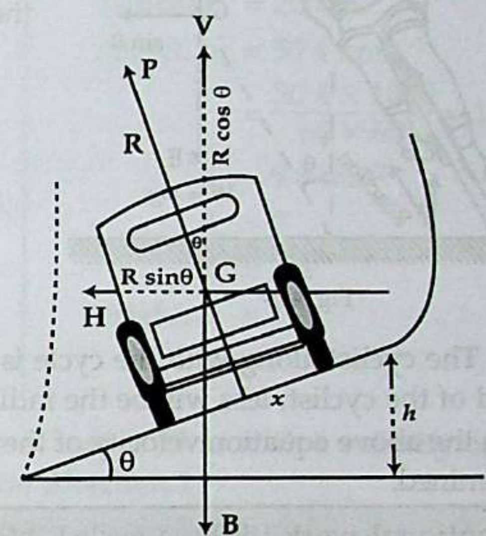


Fig. 4·36

$$R \sin \theta = \frac{mv^2}{r}$$

$$R \cos \theta = W = mg$$

Dividing the above two equations, we get

$$\tan \theta = \frac{v^2}{rg} \quad \ldots \quad \ldots \quad \ldots \quad (4.51)$$

From this equation we can determine the accurate angle of banking. This value depends on the speed of the vehicle. For this reason, for particular value of the velocity of a vehicle, accurate banking of the road can be made.

Nowadays, in order to avoid accident in every turning of a modern highway, a banking is made. Banking is also made in rails. Here outer side of the rail is made higher than the inner side. In all turning a sign board indicates the maximum allowable speed of the vehicles. So the drivers remain alert not to drive the vehicle above the quoted speed. Hence, number of accident reduces.

2. Motion of a cyclist along a curved path :

We can discuss the turning of a cyclist in the same way. During turning the cyclist automatically inclines towards inside i.e., towards the centre of the turning of the road [Fig. 4·37]. As a result, the cyclist asserts pressure on the road inclinedly, so the reaction R of the road acts making an angle θ with the horizontal. The horizontal component of the reaction provides the necessary centripetal force. If the cyclist along with the cycle bends inside making an angle θ with the vertical in order to balance in the turning, then the vertical and horizontal components of the reaction R will be R cos θ and R sin θ respectively. This vertical component of reaction balances the weight mg of the cyclist along with the cycle and the horizontal component provides the necessary centripetal force $\frac{mv^2}{r}$.

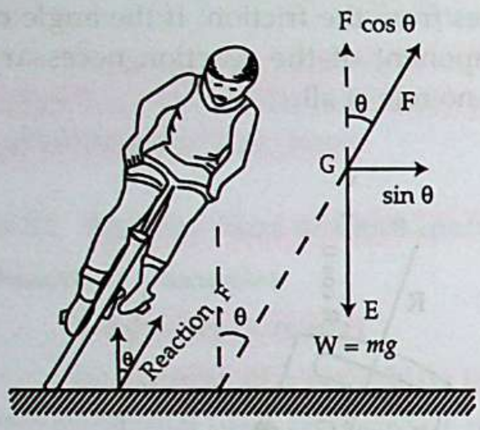


Fig. 4·37

$$\therefore \quad R\cos\theta = mg$$

$$\text{and } R\sin\theta = \frac{mv^2}{r}$$

$$\text{or, } \tan\theta = \frac{v^2}{rg}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

The cyclist along with the cycle is to take turn making this angle θ. The larger the speed of the cyclist, less will be the radius of the turning and he will have to bend more. From the above equation velocity of the cyclist along with the cycle, $v = \sqrt{rg}\tan\theta$ can be determined.

Perceptional work : Why a cyclist along with the cycle needs to incline towards the centre of circle while pedalling the cycle in a curved path ?

There is a possibility of skidding to take turn in a straight path. A centripetal force is needed along horizontal towards the centre of the circle for a cyclist along with the cycle in circular path. To provide this centripetal force the cyclist along with his cycle needs to incline towards the centre of the curvature.

3. Motion of the planets :

Planets are revolving around the sun in their own orbits. Here the gravitational force of the sun acting on each planet is the necessary centripetal force.

Similarly, for satellites revolving around the planet the centripetal force is the gravitational force of the planet.

Mathematical examples

1. At what velocity a motor cyclist should move in a circular path of diameter of 50 m so that he remains inclined at an angle of 30° with the vertical plane ?

We know,

$$\tan\theta = \frac{v^2}{rg}$$

$$\text{or, } v^2 = rg\tan\theta$$

$$\text{or, } v = \sqrt{rg\tan\theta}$$

$$\therefore \quad v = \sqrt{25 \times 9{\cdot}8 \times \tan(30°)}$$

$$= \sqrt{25 \times 9{\cdot}8 \times 0{\cdot}577}$$

$$= 11{\cdot}89 \text{ ms}^{-1}$$

Here,

$$r = \frac{50\text{ m}}{2} = 25\text{ m}$$

$$\theta = 30°$$

$$g = 9{\cdot}8 \text{ ms}^{-2}$$

$$v = ?$$

2. In order to drive a car at a speed of 50·4 kmh^{-1} along a path of radius of banking of 200 m, at what angle the path should remain inclined ? If the width of the road is 2m, then what should be the height of the outer edge of the road to that of the inner edge ?

We know,

$$\tan\theta = \frac{v^2}{rg}$$

or, $$\tan\theta = \frac{(14)^2}{200 \times 9{\cdot}8} = 0{\cdot}1$$

$$\therefore \quad \theta = \tan^{-1}(0{\cdot}1) = 5{\cdot}7^\circ$$

When θ is small, tan θ can be written as,

$$\tan\theta = \sin\theta = \frac{h}{x}$$

$$\therefore \quad 0{\cdot}1 = \frac{h}{2}$$

$$\therefore \quad h = 0{\cdot}1 \times 2 = 0{\cdot}2 \text{ m}$$

Here,

radius, $r = 200$ m

speed, $v = 50{\cdot}4$ kmh^{-1}

$$= \frac{50{\cdot}4 \times 1000}{60 \times 60} = 14 \text{ ms}^{-1}$$

$g = 9{\cdot}8$ ms^{-2}

$\theta = ?$

$x = 2$m

height, $h = ?$

3. A cyclist is moving in a circular path of radius 18 m with a velocity of 20 km per hour. What is its slope towards the vertical direction ?

Let the slope of the cyclist towards the vertical direction be θ.

We know,

$$\tan\theta = \frac{v^2}{rg}$$

$$\therefore \quad \tan\theta = \frac{(5{\cdot}55)^2}{18 \times 9{\cdot}8} = 0{\cdot}1746$$

$$\therefore \quad \theta = \tan^{-1}(0{\cdot}1746) = 9{\cdot}9^\circ$$

Here,

velocity of the cyclist,

$$v = 20 \text{ kmh}^{-1} = \frac{20 \times 1000}{60 \times 60} \text{ ms}^{-1} = 5{\cdot}55 \text{ ms}^{-1}$$

radius of the circular path,

$r = 18$ m

$g = 9{\cdot}8$ ms^{-2}

4·32 Collision

Sudden and large change of motion of a body by a force acting for a very short duration is called collision. Striking a cricket ball by a bat, striking a carom globule by a striker, firing a bullet from a gun, etc are examples of collision. When an alpha particle approaches a gold nucleus then they repel each other with a high speed for a very short time. This event is called collision.

Collisions are of two types, viz.—

(a) Elastic Collision and

(b) Inelastic Collision

4·32·1 Elastic collision

When collision occurs between atoms or molecules and electrons, protons, neutrons etc. then total kinetic energy remains constant. This type of collision may be considered as total elastic collision. This type of collision is an ideal event. In reality such collision is not seen. So we can say that if **relative velocity of two bodies before and after collision remains unchanged then that collision is called total elastic collision.** The total kinetic energy before collision remains same after the collision.

Example : Collision between two balls of glass or steel is almost an elastic collision.

Again, if after the collision the two bodies instead of being attached to each other gets separated from each other, then that collision is called **partial elastic collision.** Total kinetic energy is not conserved in this type of collision also. During collision some of the energy is changed into another energy, mostly as heat energy. In reality, this type of collision occurs.

In case of total elastic collision :

Suppose two bodies of masses m_1 and m_2 moving along a same straight line and in the same direction with initial velocity v_{01} and v_{02}, respectively collided head on. Suppose the collision is total elastic [Fig. 4·38].

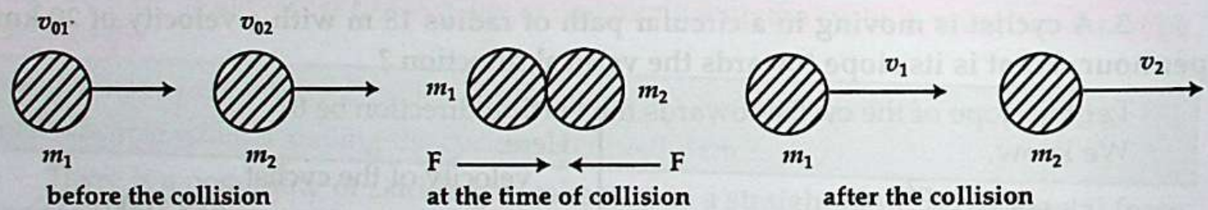


Fig. 4·38 : Total elastic collision.

During collision the two bodies apply action and reaction force F on each other. F is an impulse and due to its action momentum of each body is changed. Suppose, the two bodies after collision started moving along the same direction with velocities v_1 and v_2 respectively. It has been assumed that the direction of motion of the two bodies before and after collision is the same. Here, if $v_{01} > v_{02}$ then collision between the two bodies will occur and when $v_2 > v_1$ the two bodies will move away after collision.

Here, relative velocity before collision $= v_{01} - v_{02}$

and relative velocity after collision $= v_2 - v_1$

According to definition, $v_{01} - v_{02} = v_2 - v_1$

Now according to the conservation principle of momentum,

total momentum before collision = total momentum after collision

$$m_1v_{01} + m_2v_{02} = m_1v_1 + m_2v_2 \quad \ldots \quad \ldots \quad \ldots \quad (4.52)$$

From this equation values of v_1 and v_2 can be determined. From these two velocities kinetic energy after collision may also be found out.

Again, total kinetic energy before collision $= \frac{1}{2} mv_{01}^2 + \frac{1}{2} mv_{02}^2$ is conserved in case of total elastic collision.

So, by taking kinetic energies before and after collision equal, we get,

$$\frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2 = \frac{1}{2} mv_{01}^2 + \frac{1}{2} mv_{02}^2 \quad \cdots \quad \cdots \quad (4.53)$$

That means, kinetic energy after collision = kinetic energy before collision.

Determination of velocity after collision :

Let two bodies of mass as of m_1 and m_2 move along the same straight line with velocities v_{01} and v_{02} respectively. Since $v_{01} > v_{02}$ so elastic collision takes place between them. If the velocities after collision are v_1 and v_2, then according to the conservation principle of momentum,

total momentum before collision = total momentum after collision

$$m_1 v_{01} + m_2 v_{02} = m_1 v_1 + m_2 v_2$$

$$m_1 (v_{01} - v_1) = m_2 (v_2 - v_{02}) \quad \cdots \quad \cdots \cdots \quad (i)$$

Since the collision is elastic so summation of the kinetic energies before the collision will be equal to the summation of kinetic energies after collision i.e.,

$$\frac{1}{2} m_1 v_{01}^2 + \frac{1}{2} m_2 v_{02}^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\frac{1}{2} m_1 (v_{01}^2 - v_1^2) = \frac{1}{2} m_2 (v_2^2 - v_{02}^2) \quad \cdots \quad \cdots \quad (ii)$$

by dividing equation (ii) by (i) we get,

$$v_{01} + v_1 = v_2 + v_{02}$$

$$v_2 = v_{01} + v_1 - v_{02}$$

by inserting the value of v_2 in equation (i) we get,

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} v_{01} + \frac{2m_2}{m_1 + m_2} v_{02} \quad \cdots \quad \cdots \quad \cdots \quad (iii)$$

by inserting the magnitude of v_1 in equation (i) we get (after calculation)

$$v_2 = \frac{m_2 - m_1}{m_1 + m_2} v_{02} + \frac{2m_1}{m_1 + m_2} v_{01} \quad \cdots \quad \cdots \quad \cdots \quad (iv)$$

Special cases :

(i) **if the mass of the two bodies are equal** i.e., $m_1 = m_2$, then from equation (iii) and (iv) we get, $v_{01} = v_2$ and $v_{02} = v_1$, i.e., after collision the two bodies exchange velocities.

(ii) **masses of two bodies are equal and at the start second body is at rest.** Here $m_1 = m_2$ and $v_{02} = 0$. So we get from the above two equations, $v_1 = 0$ and $v_2 = v_{01}$.

That is after collision the first body stops and the second body continues to move with velocity of the first body.

(iii) **Masses of the two bodies are unequal and initially the second body is motionless** Here $m_1 \neq m_2$ and $v_{02} = 0$. From equations (iii) and (v) we get,

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} v_{01} \quad \text{and} \quad v_2 = \frac{2m_1}{m_1 + m_2} v_{02}$$

That is, if $m_1 \neq m_2$ then due to collision first body cannot be made motionless.

(iv) **First body is very heavy and at the start the second body is motionless.** Here $m_1 >> m_2$ and $v_{02} = 0$. So it can be written, $m_2 - m_1 \simeq -m_2$ and $m_2 + m_1 \simeq m_2$. So, we get, $v_1 = v_{01}$ and $v_2 = 2v_{01}$. That means, after collision the heavy body will remain almost stationary and the light body will speed up almost with double velocity.

(v) **The second body is very heavy and initially motionless.** Here, $m_2 >> m_2$ and $v_{02} = 0$. So it can be written, $m_1 - m_2 \simeq -m_2$ and $m_1 + m_2 \simeq m_2$. From equation (iii) and (iv) we get, $v_1 = v_{01}$ and $v_2 = 0$. That means after collision the heavy body will remain stationary and the light body will speed up in opposite direction with initial velocity.

Perceptional work : Why a ball rebounds backward after colliding with a wall ?

In case of elastic collision between two bodies, velocity of the first body after collision, $v_1 = \frac{m_1 - m_2}{m_1 + m_2} v_{01} + \left(\frac{2m_1}{m_1 + m_2}\right) \times v_{02}$ for the collision of the ball with the wall, $v_{02} = 0$ and $m_2 >> m_1$. So, $v_1 = -v_{01}$ and $v_{02} = 0$ i.e., the ball will remain static and the ball will return in the opposite direction with same velocity.

Practical work : Take a ball in your hand. Now throw it on a table. Which direction will the table move ? Why the ball will come back after collision with the table ?

When a lighter body collides with a heavier body, the heavier body remains at rest and the lighter body moves away in the opposite direction with double velocity.

Perceptional work : In case of elastic collision two bodies of equal mass mutually exchange velocities—explain.

Two bodies of equal mass after collision mutually exchange velocities. Its explanation in given below :

Suppose the masses of the two bodies are $m_1 = m_2$ and velocities after elastic collision are v_1 and v_2 and velocities before collision are u_1 and u_2. So, we get,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

or, $m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$

or, $u_1 - v_1 = v_2 - u_2$ ... ... (i)

Again, from equation (i) we get, $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

or, $\frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_2u_2^2$

or, $u_1^2 - v_1^2 = v_2^2 - u_2^2$ ($\because m_1 = m_2$) ... (ii)

by dividing equation (ii) by equation (i) we get,

$$\frac{u_1^2 - v_1^2}{u_1 - v_1} = \frac{v_2^2 - u_2^2}{v_2 - u_2}$$

or, $\frac{(u_1 - v_1)(u_1 + v_1)}{u_1 - v_1} = \frac{(v_2 - u_2)(v_2 + u_2)}{v_2 - u_2}$

or, $u_1 + v_1 = v_2 + u_2$ ... ... (iii) by adding equation (i) and (iii) we get,

$$u_1 - v_1 + u_1 + v_1 = v_2 - u_2 + v_2 + u_2$$

or, $2u_1 = 2v_2$

$\therefore \quad \boxed{u_1 = v_2} \quad \ldots \quad \ldots \quad$ (iv)

So, two bodies of equal mass mutually exchange their velocities after elastic collision.

4·32·2 Inelastic collision

You just take two balls made of soft mud and make a collision between the two balls. You will see that total kinetic energy does not remain conserved in this collision. This type of collision is also an ideal event. It is an example of inelastic collision. That means, **after collision between two bodies if they combine with each other and move as a single body i.e., the relative velocities of the two bodies become zero, then it is called inelastic collision.**

In figure 4·39 a total inelastic collision has been shown. Here two bodies of masses m_1 and m_2 moving along the same straight line and in the same direction with velocities v_{0_1} and v_{0_2}, respectively collide with each other. After collision the two bodies combine with each other and start moving in the same direction as a single body with velocity v.

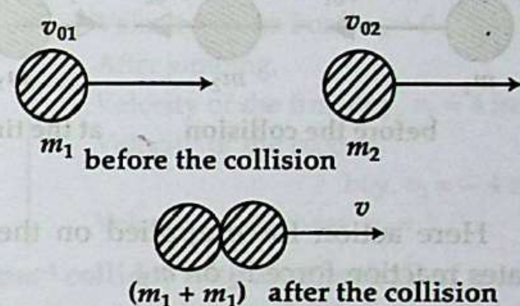


Fig. 4·39 : Total inelastic collision.

Now, from the principle of conservation of linear momentum, we get,

Total momentum before collision = Total momentum after collision.

$$\therefore \quad m_1 v_{01} + m_2 v_{02} = (m_1 + m_2)v$$

$$\text{or,} \quad v = \frac{m_1 v_{01} + m_2 v_{02}}{m_1 + m_2} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (4.54)$$

By subtracting the total kinetic energy before collision $\frac{1}{2} m v_{02}^2 + \frac{1}{2} m_2 v_{02}^2$ from the total kinetic energy after collision $\frac{1}{2}(m_1 + v_2)v^2$, we can find dissipation of kinetic energy. It is seen that dissipation of kinetic energy is proportional to the square of relative velocity $(v_{01} - v_{02})$.

Verify : Look at the adjacent picture [Fig. 4·40]. What type of collision has occurred between the two cars ? Naturally, the relative velocity after collision is zero. If the velocities of the two cars before collision were u_1 and u_2 respectively, then write and express the equation for velocity v after collision.

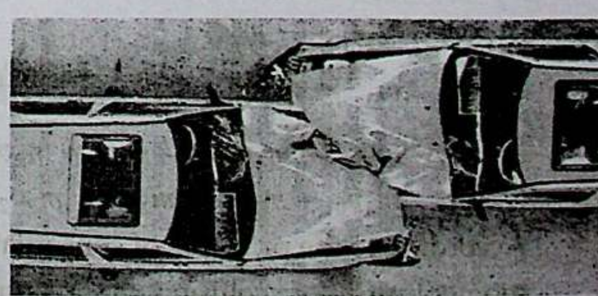


Fig. 4·40

4·32·3 One-dimensional collision

When the children play marbles, then if one marble collides with another marble and after the collision if the marbles move in a straight line, then that collision is called one-dimensional collision. **That means, if the relative motion of the two colliding bodies is along the same straight line before and after the collision, then that collision is called one dimensional collision.**

Suppose two particles of masses m_1 and m_2 are moving in a straight line along X-axis with velocities v_{01} and v_{02} respectively [Fig. 4·41]. Here $v_{01} > v_{02}$. At one time the first body hits the second body from behind and afterwards the two particles move along the same straight line in the same direction with velocities v_1 and v_2 respectively.

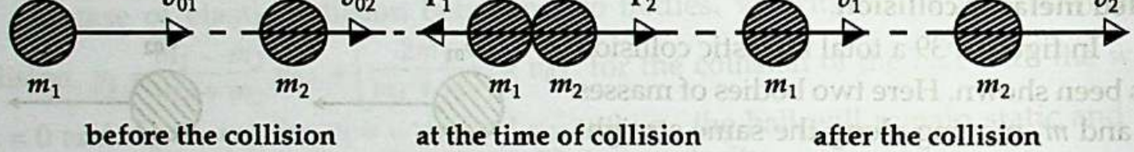


Fig. 4·41

Here action force applied on the body of mass m_2 is F_1 and the mass m_2 also eates reaction force F_2 on m_1.

Again, suppose if the duration of the action and reaction is t, then

sum of initial momentum of the two particles $= m_1v_{01} + m_2v_{02}$ ... (4.55)

and the sum of the final momentum of the two particles $= m_1v_1 + m_2v_2$

According to Newton's third law of motion,

action = reaction

$\therefore\ F_2 = -F_1$

During collision the action and reaction forces act for the same duration.

Suppose, the two bodies move along the same straight line, before and after the collision, with velocities v_{01} and v_{02} respectively. Due to action and reaction, accelerations of the two bodies become a_1 and a_2.

$$\therefore\quad F_1 = -F_2$$

$$\text{or,}\quad m_1a_1 = -m_2a_2$$

$$\text{or,}\quad m_1\frac{(v_1 - v_{01})}{t} = -m_2\frac{(v_2 - v_{02})}{t}$$

$$\text{or,}\quad m_1v_1 - m_1v_{01} = -m_2v_2 + m_2v_{02}$$

$$\text{or,}\quad m_1v_{01} + m_2v_{02} = m_1v_1 + m_2v_2 \quad \ldots \quad \ldots (4.56)$$

According to the principle of conservation of momentum,

momentum before collision = momentum after collision.

This is the equation of one-dimensional collision.

Mathematical examples

1. Two boys are standing on two ends of a boat of mass of 200 kg which is floating on water at rest. Their masses are 40 kg and 70 kg respectively. Each of them jumps simultaneously from the boat with a horizontal velocity of 4 ms^{-1}. Then at what velocity and in which direction will the boat be moving ?

Given,

Mass of the first boy, $m_1 = 40$ kg
Mass of the second boy, $m_2 = 70$ kg
Mass of the boat, $m_3 = 200$ kg
Before jumping,
Velocity of the first boy, $u_1 = 0$
Velocity of the second boy, $u_2 = 0$
Velocity of the boat, $u_3 = 0$
After jumping,
Velocity of the first boy, $v_1 = 4$ ms^{-1}
Velocity of the second boy, $v_2 = -4$ ms^{-1}
Velocity of the boat, $v_3 = ?$

We know,

$$m_1u_1 + m_2u_2 + m_3u_3 = m_1v_1 + m_2v_2 + m_3v_3$$

or, $0 + 0 + 0 = 40 \times 4 + 70 \times -4 + 200 \times v_3$

or, $-120 + 200\, v_3 = 0$

$\therefore$ $v_3 = 0{\cdot}6$ ms^{-1} and its direction will be towards m_2

2. A ball of mass of 2 kg with velocity of 3 ms^{-1} collides with another ball of mass of 0·5 kg at rest. If (a) these balls attach together after collision and (b) the collision is total elastic, then what will be the velocities of the two balls after collision ?

(a) Since the two balls combine with each other after collision, so the collision is total inelastic. Here, the velocity of the two balls will be same after collision.

According to the principle of conservation of momentum,

$$m_1v_{01} + m_2v_{02} = m_1v_1 + m_2v_2$$

So, in this case if we put $v_1 = v_2 = v$ and $v_2 = 0$, then we get,

$$2 \times 3 + 0 = (2 + 0{\cdot}5) \times v$$

$\therefore$ $v = 2{\cdot}4$ ms^{-1}

(b) Since the collision is total elastic, we know in case of total elastic collision,

$$v_{01} - v_{02} = v_2 - v_1$$

$$3 - 0 = (v_2 - v_1)$$

$\therefore$ $v_2 - v_1 = 3$ ms^{-1} ... ... (i)

Again, according to the principle of conservation of momentum,

$$m_1v_{0_1} + m_2v_{0_2} = m_1v_1 + m_2v_2$$

$$2 \times 3 + 0 = 2v_1 + 0{\cdot}5v_2$$

$\therefore$ $2v_1 + 0{\cdot}5v_2 = 6$ ... ... (ii)

Solving (i) and (ii) we get, $v_1 = 18$ ms^{-1} and $v_2 = 4{\cdot}8$ ms^{-1}

So, the two balls will move in the same direction after collision.

3. A ball of mass 2 kg engaged directly in elastic collision at velocity of 3 ms^{-1} with a stationary body of mass 0·5 kg. What will be the velocity of the second body after collision ? [Ch. B. 2015]

We know,

$$v_2 = \frac{m_1 - m_2}{m_1 + m_1} \times v_{02} + \frac{2\,m_1}{m_1 + m_2} \times v_{01}$$

$$= \left(\frac{2 - 0{\cdot}5}{2 + 0{\cdot}5}\right) \times 0 + \frac{2 \times 2}{2 + 0{\cdot}5} \times 3$$

$$= 4{\cdot}8 \text{ ms}^{-1}$$

Here,

$m_1 = 2$ kg

$v_{01} = 3$ ms^{-1}

$v_{02} = 0$

$v_2 = ?$

4. A body of mass 4 kg is engaged in elastic collision with a stationary body. After the collision the body started moving in the same direction with velocity of one-fourth of the initial velocity. What is the mass of the stationary body ?

We know,

$$v_2 = \frac{m_1 - m_2}{m_1 + m_2} \times v_{02} + \frac{2\,m_1}{m_1 + m_2} \times v_{01}$$

or, $$v_2 = \frac{4 - m_2}{4 + m_2} \times v_{02} + \frac{2\,m_1}{m_1 + m_2} \times 0$$

or, $$\frac{v_2}{v_{02}} = \frac{4 - m_2}{4 + m_2}$$

or, $$\frac{1}{4} = \frac{4 - m_2}{4 + m_2}$$

or, $4 + m_2 = 16 - 4m_2$

or, $5m_2 = 12$

$\therefore \quad m_2 = \frac{12}{5}$ kg $= 2{\cdot}4$ kg

Here,

$m_1 = 4$ kg

$m_2 = ?$

$\frac{v_2}{v_{02}} = \frac{1}{4}$

$v_{01} = 0$

Necessary mathematical formulae

$\omega = \frac{\theta}{t}$ ... ... (1)

$\omega = \frac{d\theta}{dt}$ ... ... (2)

$\alpha = \frac{\omega_2 - \omega_1}{t}$ ... ... (3)

$\alpha = \frac{d\omega}{dt}$ ... ... (4)

$a = \alpha r$ ... ... (5)

$\theta = \omega t$ ... ... (6)

$\omega = \omega_0 + \alpha t$ ... ... (7)

$\theta = \omega_0 + \frac{1}{2}\alpha t^2$ ... ... (8)

$\omega = \frac{\theta}{T} = \frac{2\pi}{T} = \frac{2\pi\omega}{t} = 2\pi n = \frac{v}{t}$ ... ... (9)

$$\alpha = \frac{\omega}{t} \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

$$F = ma \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} v_{01} + \left(\frac{2m_1}{m_1 + m_2}\right) \times v_{02} \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

$$F = \frac{mv^2}{r} = m\omega^2 r \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

$$\tan\theta = \frac{v^2}{rg} \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

$$I = \sum mr^2 = MK^2 \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

$$K.E = \frac{1}{2} I\omega^2 \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

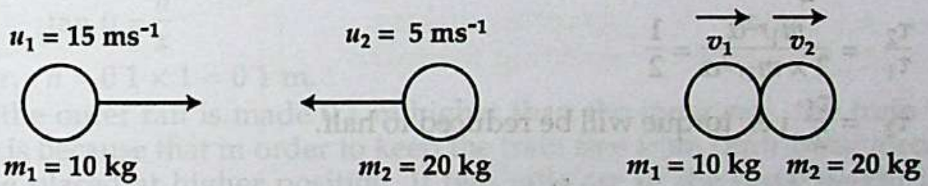
$$L = I\omega \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

$$\tau = I\alpha \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

$$\tau = \frac{dL}{dt} \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

Higher efficiency mathematical examples

1. Look at the following figure. (a) As per the event mentioned in the stimulus what will be the combined velocity ? (b) Will the kinetic energy be conserved ? —Explain.



before collision **after collision**

(a) According to the figure, we know,

$$m_1u_1 - m_2u_2 = (m_1 + m_2)v$$

$$10 \times 15 - 20 \times 5 = (10 + 20)(v)$$

$$\text{or, } 150 - 100 = 30 \times v$$

$$\text{or, } 50 = 30\,v$$

$$\therefore v = \frac{5}{3} \text{ ms}^{-1}$$

(b) Kinetic energy before collision $= \frac{1}{2} mu_1^2 + \frac{1}{2} mu_2^2$

$$= \frac{1}{2} \times 10 \times (15)^2 + \frac{1}{2} \times 20 \times (5)^2$$

$$= 5 \times (15)^2 + 10 \times 25$$

$$= 1125 + 250 = 1375 \text{ J}$$

Kinetic energy after collision $= \frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2$ $[\because v_1 = v_2 = v = \frac{5}{3} \text{ ms}^{-1}]$

$$= \frac{1}{2} \times 10 \times \left(\frac{5}{3}\right)^2 + \frac{1}{2} \times 20 \times \left(\frac{5}{3}\right)^2$$

$$= 5 \times \frac{25}{9} + 10 \times \frac{25}{9}$$

$$= 13{\cdot}89 + 27{\cdot}78 = 41{\cdot}67 \text{ J}$$

It is seen that in both the cases kinetic energy is not same. That means, kinetic energy is not conserved.

2. A body of mass of 4 kg is fastened with a long rope of length 0·2 m and is rotated around a fixed axis with angular velocity 2 rad s^{-1}.

(a) Calculate the angular momentum of the rotating body.

(b) What change of torque will be if the mass of the body is made half ? Explain by mathematical analysis. [J. B. 2016]

(a) We know,

$$L = I\omega = mr^2\omega = 8 \times (0{\cdot}2)^2 \times 2 = 0{\cdot}64 \text{ kg m}^2\text{s}^{-1}$$

((b) We know, if angular acceleration is α, then torque, $\tau = I\alpha = mr^2\alpha$

$\therefore \quad \tau_1 = m_1 r^2 \alpha$

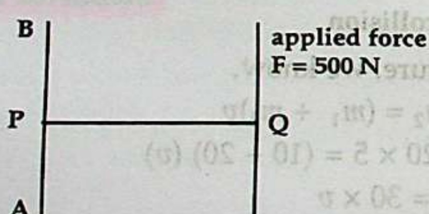
or, if $m_2 = \frac{m_1}{2}$, then

or, $\tau_2 = \frac{m_1}{2} r^2 \alpha$

$\therefore \quad \frac{\tau_2}{\tau_1} = \frac{m_1 r^2 \alpha}{2 \times m_1 r^2 \alpha} = \frac{1}{2}$

$\therefore \quad \tau_2 = \frac{\tau_1}{2}$ i.e., torque will be reduced to half.

3.



(a) Calculate the torque of the rod PQ rotating with respect to the rotation axis AB.

(b) If the rotation axis AB from the end of the rod PQ is taken to the mid point, then in which case moment of inertia will be more—give mathematical justification in favour of your answer. [S. B. 2016]

(a) We know,

torque, $\tau = rF \sin\theta$

$= 1 \times 500 \times \sin 90°$

$= 500$ N

(b) moment of inertia with respect to the axis passing through the end of a rod,

$$I_1 = \frac{Ml^2}{3}$$

Again, if the rotation axis passes through the centre of the rod then,

$$I_2 = \frac{Ml^2}{12}$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{Ml^2}{3} \times \frac{12}{Ml^2} = 4$$

$\therefore \quad I_1 = 4I_2$ i.e., in the first case moment of inertia will be more.

4. A train is turning along the bending of a rail line of radius of 200 m. Distance between two plates is 1 m. For turning the running train at velocity at 50·4 km.

(a) Calculate the of the inner and outer plates of the rain line and

(b) What will happen if the height of the two plates are equal and if not equal then what will happen ? explain.

(a) We know,

$$\tan\theta = \frac{v^2}{rg}$$

$$\text{or,} \quad \tan\theta = \frac{(14)^2}{200 \times 9{\cdot}8} = 0{\cdot}1$$

$$\therefore \quad \tan\theta = 0{\cdot}1$$

Here,

$r = 200$ m

$g = 9{\cdot}8$ ms^{-2}

$v = 50{\cdot}4$ km h^{-1} $= \frac{50{\cdot}4 \times 1000}{60 \times 60}$

$= 14$ ms^{-1}

$r = 1$ m

When θ is small, then tan θ = 0·1 can be written.

Suppose the distance between the two rails = x.

If the height of one line with respect to the other is h, then

$$\tan\theta = \frac{h}{x}$$

$$\text{or,} \quad h = 0{\cdot}1 \times 1 = 0{\cdot}1 \text{ m.}$$

(b) If the outer rail is made 0·1 m higher than the inner rail, the train will move freely. This is because that in order to keep the train free from centripetal force the outer rail must be placed at higher position. If two rails are of the same height then while taking turn necessary centripetal force is to be applied. Due to the lack of the centripetal force the train may skid because of moment of inertia. In order to balance this moment of inertia the outer rail is made higher than the inner rail.

5. A small body A of mass 150 g is allowed to rotate by a thread of 75 cm of length. It starts rotating from rest and after 3 minutes it rotates 120 times per minute.

(a) Calculate the effective torque on the body A.

(b) What amount of tension will act on the body A ? If the magnitude of tension is made 4 times, what change of linear velocity will take place —give opinion.

(a) We know,

$$\omega = 2\pi n = 2\pi \times 2$$

$$= 12{\cdot}56 \text{ rad s}^{-1}$$

If angular acceleration is α, then

$$\omega = \omega_0 + \alpha t$$

$$\text{or,} \quad 12{\cdot}56 = 0 + \alpha \times 3 \times 60$$

$$\therefore \quad \alpha = 0{\cdot}0698 \text{ rad s}^{-1}$$

Here,

$m = 150$ g $= 0{\cdot}15$ kg

$r = 75$ cm $= 0{\cdot}75$ m

$t = 3$ min $= 3 \times 60$ s $= 180$ s

$n = 120$ rev/min.

$n = \frac{120}{60}$ rev/s $= 2$ rev/s

moment of inertia, $I = mr^2 = 0.15 \times (0.75)^2 = 0.0844$ kgm^2

torque, $\tau = I\alpha = 0.0844 \times 0.0698 = 5.89 \times 10^{-3}$ Nm

(b) mass of the body, $m = 150$ g $= 0.15$ kg

$r = 75$ cm $= 0.75$ m

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 120}{60} = 12.56 \text{ rad s}^{-1}$$

linear velocity, $v = \omega r = 12.56 \times 0.75 = 9.42$ ms^{-1}

tension of the thread, $F = \frac{mv^2}{r} = \frac{0.15 \times (9.42)^2}{0.75} = 17.75$ N

Hence, tension on the body that will act is 17.75 N

Again, if the tension is 4 times, then $F_1 = 4F$; in that case if linear velocity is v_1, then $F_1 = \frac{mv_1^2}{r}$ and $F = \frac{mv^2}{r}$

$$\therefore \quad \frac{F_1}{F} = \frac{v_1^2}{v^2} = \left(\frac{v_1}{v}\right)^2 = 4$$

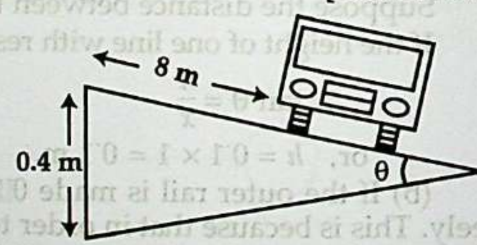
$$\therefore \quad \frac{v_1}{v} = 2 \text{ or, } v_1 = 2v$$

i.e., if tension is 4 times, then linear velocity will be double.

6. While taking turn in a curvature of radius of 100 m with velocity of 30 kmh^{-1} a bus skidded from the road to a ditch. [Ch. B. 2016]

(a) Determine the banking angle of the mentioned road in the stimulus.

(b) In the light of the stimulus analyse mathematically the reason of skidding the bus in the ditch.



(a) We know,

if the value of θ is very small, then

$$\tan\theta = \sin\theta = \frac{h}{d}$$

$$\therefore \quad \theta = \sin^{-1}\frac{h}{d} = \sin^{-1}\frac{0.4}{8} = 2.86°$$

(b) If for safe driving the car the banking angle is θ, then

$$\tan\theta = \frac{v^2}{rg}$$

$$\text{or, } \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$= \tan^{-1}\left\{\frac{(8.33)^2}{100 \times 9.8}\right\} = 4.05°$$

Here,

$v = 30$ kmh^{-1}

$= \frac{30 \times 1000}{60 \times 60}$ ms^{-1} $= 8.33$ ms^{-1}

$r = 100$ m

The banking angle of the road in the stimulus is 2.86° but for safe driving in that path at velocity of 30 kmh^{-1} the banking angle should have been 4.05°. So, the car skidded.

7. A circus player was rotating a body over his head in the vertical plane. The length of the thread was 90 cm and the body was revolving 100 times per minute. All on a sudden one third of the rotating body was torn off. Due to this the player, instead of becoming nervous, increased the length of the thread to keep the number of revolution per minute unchanged.

(a) What was the centripetal acceleration of the body before its mass was decreased ?

(b) Verify the correctness of the change of length of the thread made by the player by mathematical analysis.

(a) We know,

angular veolocity, $\omega = \frac{2\pi N}{t} = \frac{2\pi \times 100}{60}$

$= 10.472 \text{ rad s}^{-1}$

and centripetal acceleration,

$a = \omega^2 r = (10.472)^2 \times 0.9 = 98.7 \text{ ms}^{-2}$

Here,

N = 100 times

$t = 1 \text{ min} = 60 \text{ s}$

$r = 90 \text{ cm} = 0.9 \text{ m}$

(b) Tension or centripetal force applied by the hand of the player remained unchanged.

Centripetal force or tension of the thread, $F_C = ma = m \times 98.7 = 98.7 \text{ mN}$

When the mass is reduced by one third

then the remaining mas m', $= m - \frac{m}{3} = \frac{2m}{3}$

In this case, if the new length of the thread is r', then

$m'\omega^2 r' = m\omega^2 r$

or, $m'r' = mr$

or, $r' = \frac{mr}{m'} = \frac{mr}{2m/3} = \frac{3}{2}r$

So, change of length $= \frac{3r}{2} - r = \frac{r}{2}$

or, length was increased by 50% of the previous length.

8. Two bodies of masses 40 kg and 60 kg while coming from opposite direction of each other with velocities of 10 ms^{-1} and 5 ms^{-1} respectively collided with one another. After the collision the two bodies combinedly continued to move.

(a) What was the velocity of the combined body.

(b) Why the collision was not elastic ? Give your opinion with mathematical analysis.

(a) We know,

$m_1u_1 + m_2u_2 = (m_1 + m_2)\, v$

or, $40 \times 10 - 60 \times 5 = (40 + 60)\, v$

$\therefore \quad v = \frac{100}{100} = 1 \text{ ms}^{-1}$

direction along the velocity 10 ms^{-1}

Here,

$m_1 = 40 \text{ kg}$

$m_2 = 60 \text{ kg}$

$u_1 = 10 \text{ ms}^{-1}$

$u_2 = -5 \text{ ms}^{-1}$

v = velocity of the combined body = ?

(b) We know, during elastic collision both momentum and kinetic energy remain conserved; but in inelastic collision although momentum remains conserved but kinetic energy is not conserved. Kinetic energy is transformed into another energy.

Total energy of the two bodies before collision,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2} \times 40 \times 10^2 + \frac{1}{2} \times 60\,(-5)^2$$

$$= (2000 + 750)\text{ J} = 2750\text{ J}$$

After collision kinetic energy of the combined two bodies,

$$\frac{1}{2}(m_1 + m_2)\,v^2 = \frac{1}{2} \times 100 \times (1)^2 = 50\text{ J}$$

In this case kinetic energies before and after the collision are not conserved. Hence the collision is not elastic.

9. Radius of the bend of a road is 50 m and difference of height of both sides is 0·5 m; width of the road is 5 m.

(a) What is the actual banking angle of the road ?

(b) Whether a car can move safely on the road of the stimulus velocity of 108 kmh^{-1} is possible to move on the road of or not ?—verify mathematically.

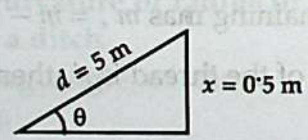
[R. B. 2017]

(a) We know,

$$\sin\theta = \frac{x}{d} = \frac{0{\cdot}5}{5}$$

$$\therefore\quad \theta = \sin^{-1}\left(\frac{0{\cdot}5}{5}\right)$$

$$= 5{\cdot}74°$$



Here,

width of the road, $d = 5$ m

difference of height of both sides, $x = 0{\cdot}5$ m

banking angle, $\theta = ?$

(b) From (a) we get, $\theta = 5{\cdot}74°$

According to the stimulus, radius of the bend of the road,

$r = 50$ m

If the maximum velocity of the car is v, then

$$\tan\theta = \frac{v^2}{rg}$$

$$\therefore\quad v^2 = \tan\theta \times rg$$

$$\text{or,}\quad v = \sqrt{\tan\theta \times rg}$$

$$= \sqrt{\tan(5{\cdot}74) \times 50 \times 9{\cdot}8}$$

$$= 7{\cdot}02\text{ ms}^{-1}$$

$$= 25{\cdot}27\text{ kmh}^{-1}$$

That means, in this road maximum velocity at which a car can move safely is 25·27 kmh^{-1}.

So, on the road of the stimulus it is not possible to move a car safely at velocity of 108 kmh^{-1}.

10. Nayan is rotating a piece of stone of mass 25 g by fastening at the end of a thread of length of 1 m. The piece of stone is rotating 5 times per second. Keeping the number of rotation constant, the length of the thread is made double. The thread can withstand a force of 40 N.

(a) Calculate the angular momentum of the stone in the first case.

(b) Whether Nayan will be able to rotate the stone successfully by increasing the length double or not—verify mathematically. [Din. B. 2017]

(a) We know, angular momentum,

$$L = mvr = mr^2\omega$$

$$= mr^2 \times \frac{2\pi N}{t} \quad \left[\because \omega = \frac{2\pi N}{t}\right]$$

$$= \frac{25 \times 10^{-3} \times (1)^2 \times 2 \times 3^{\cdot}1416 \times 5}{1}$$

$$= 0^{\cdot}7854 \text{ kgm}^2\text{s}^{-1}$$

Here,
length of the thread, $r = 1$ m
mass of the piece of stone, $m = 25$ g $= 25 \times 10^{-3}$ kg
time, $t = 1$ sec
angular momentum, L = ?

(b) Changed length of the thread i.e., changed radius,

$r = 2 \times 1 = 2$ m

So, maximum tolerable force, F = 40 N

$\therefore$ angular velocity, $\omega = \frac{2\pi N}{t}$

$$= \frac{2 \times 3^{\cdot}1416 \times 5}{1}$$

$$= 31^{\cdot}416 \text{ rads}^{-1}$$

Givven,
mass of pieece of stone, $m = 25$ g $= 25 \times 10^{-3}$ kg
number of rotation, N = 5
time, $t = 1$ sec

$\therefore$ centripetal force, $F = m\omega^2 r = 25 \times 10^{-3} \times (3^{\cdot}1416)^2 \times 2$

$= 49^{\cdot}348$ N

$\therefore$ centripetal force or tension of the thread F is greater than the maximum toterable force F.

So, Nayan will not be able to rotate the stone safely by increasing the length double. As the tension of the thread is more so the thread will tear off.

11. Distance between the two plates of the rail lines of the metre gauge and the broad gauge are respectively 0˙8 m and 1˙3 m. The place where the radius of the bend is 500 m, differences of height of the lines at that place are respectively 7˙00 cm and 11˙37 cm.

(a) What is the banking angle of the first line ?

(b) In which line train can take turn speedily ? Give opinion with mathematical analysis. [S. B. 2017]

(a) We know,

$$\tan\theta = \frac{h}{l} = \frac{0^{\cdot}07}{0^{\cdot}8} = 0^{\cdot}0875$$

$\therefore$ $\theta = \tan^{-1}(0^{\cdot}0875) = 5°$

$\therefore$ banking angle of the first line = 5°

Here,
height, $h = 7^{\cdot}00$ cm $= 0^{\cdot}07$ m
distance between the two lines of the metre gauge, $l = 0^{\cdot}8$ m
banking angle, θ = ?

(b) banking of the second line,

$$\theta_2 = \tan^{-1}\left(\frac{h'}{l'}\right)$$

$$= \tan^{-1}\left(\frac{0.1137}{1.3}\right) = 5°$$

Here,
radius of the bend, $r = 500$ m
acceleration due to gravity,
$g = 9.8$ ms^{-2}
banking angle of the first line,
$\theta_1 \doteq 5°$
distance between the two lines of the broad gauge, $l' = 1.3$
difference of height,
$h' = 11.37$ cm $= 0.1137$ m

Again, if the maximum velocity of the train in the first line is v_1 and that in the second line is v_2, then

$$\tan\theta_1 = \frac{v_1^2}{rg} \text{ and } \tan\theta_2 = \frac{v_2^2}{rg}$$

$$\therefore \frac{\tan\theta_1}{\tan\theta_2} = \frac{v_1^2}{v_2^2}$$

Since, $\theta_1 = \theta_2$ so $v_1 = v_2$. That means, in both the lines the train can take turn with same speed.

12. A dancer of mass of 60 kg can rotate 20 times per minute by stretching her two hands. She tries to tune to a music.

(a) Compare the two moment of inertia if the dancer rotates 30 times per unit in order to be harmonic with the music.

(b) Will the changed angular kinetic energy of the dancer of the stimulus be double ? Give opinion analytically. [B. B. 2017]

(a) Let moment of inertia of the dancer in the first case be I_1 and angular velocity be ω_1 and in the second case moment of inertia be I_2 and angular velocity be ω_2.

Here,
In the first case,
number of rotation of the dancer per minute, $n_1 = 20$
In the second case,
number of rotation of the dancer per minute, $n_2 = 30$

$$\therefore \omega_1 = \frac{2\pi n_1}{60} = \frac{2\pi \times 20}{60} = \frac{2}{3}\pi \text{ rads}^{-1}$$

$$\text{and } \omega_2 = \frac{2\pi n_2}{60} = \frac{2\pi \times 30}{60} = \pi \text{ rads}^{-1}$$

Again, according to the conservation law of angular momentum,

$$I_1\omega_1 = I_2\omega_2$$

$$\therefore I_2 = \frac{\omega_1}{\omega_2} \times I_1 = \frac{\frac{2}{3}\pi}{\pi} \times I_1 = \frac{2}{3} I_1$$

So, moment of inertia in the second case is $\frac{2}{3}$ times the moment of inertia of the first case.

(b) angular frequency in the first case, $\omega_1 = \frac{2}{3}\pi$ rads^{-1}

moment of inertia in the first case, I_1

changed moment of inertia, $I_2 = \frac{2}{3} I_1$

So, angular kinetic energy in the first case, $E_1 = \frac{1}{2} I_1\omega_1^2$

and changed angular kinetic energy, $E_2 = \frac{1}{2} I_2\omega_2^2$

$$\therefore \quad \frac{E_1}{E_2} = \frac{\frac{1}{2} I_1\omega_1^2}{\frac{1}{2} I_2\omega_2^2} = \frac{\frac{2}{3} I_1 \times \pi^2}{I_1 \times \frac{4}{9}\pi^2}$$

$$= \frac{2}{3} \times \frac{9}{4} = \frac{3}{2} = 1{\cdot}5$$

$$\therefore \quad E_2 = 1{\cdot}5 \times E_1$$

So, changed angular kinetic energy of the dancer will not be double, rather it will be 1·5 times.

Summary

Force : The external cause which changes or tends to change the state of rest or of motion of a body is called force.

Fundamental force : Forces which are fundamental and do not originate from other forces, but all other forces are derived from these forces are called fundamental forces.

Types of fundamental forces : There are four types of fundamental forces, viz.—gravitational force, electromagnetic force, strong nuclear force and weak nuclear force.

Momentum : The property which is originated due to the combination of mass and velocity of a body is called momentum of the body. Momentum = mass × velocity.

Newton's second law of motion : Rate of change of momentum is proportional to the applied force on the body. Direction of momentum is along the direction of the applied force.

Impulsive force : Impulsive force is a force of high magnitude which acts for a very short duration.

Impulse : The product of the impulsive force and the time during which the force acts is called impulse.

Moment of inertia : The sum of the products of masses of all the particles of a rigid rotating body and the square of their respective distances from the axis of rotation about which the body rotates is called moment of inertia.

Angular momentum : Product of the radius vector of a rotating particle and the linear momentum is called the angular momentum.

Radius of gyration : It is a distance of a point from an axis of rotation such that, if a point mass equal to the mass of the body is placed at this point, its moment of inertia about the axis of rotation is equal to the moment of inertia of the whole body about the same axis.

Torque : In order to produce acceleration in a body rotating around a fixed axis the applied moment of couple is called torque or moment of force.

Conservation principle of angular momentum : If the resultant external torque acting on a body is zero, then no change of angular momentum of that body occurs. This is the conservation principle of angular momentum.

Centripetal force : When an object revolves in a circular path, the force that acts on the body in the direction of the centre of the circle so that it keeps the object in circular motion is called centripetal force.

Centrifugal force : A force equal and opposite to the centripetal force that acts on a body rotating in a circular orbit with uniform speed i.e., an imaginary force that acts outward from the centre of the circle is called the centrifugal force.

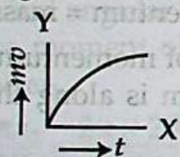
Collision : A sudden and large change of motion of a body by a large force acting for a very short duration is called collision.

Elastic collision : If relative velocities of two bodies before and after collision remains unchanged then that collision is called elastic collision.

Inelastic collision : The collision after which the two colliding bodies combine and move away as a single body i.e., the relative velocity of the two bodies is zero then it is called inelastic collision.

One-dimensional collision : If the relative velocity between two colliding bodies before and after collision is along the same straight line then it is called one-dimensional collision.

Summary of the relavant topics for the answer of multiple choice questions

1. If a car starting from rest is accelerated, then the graph of the momentum against time will be 

2. Angular velocity of an watch for hour-hand is $\frac{\pi}{720}$ rad min^{-1}, or $\frac{\pi}{21600}$ rad s^{-1}, for minute-hand is $\frac{\pi}{180}$ rad s^{-1}, for second-hand is $\frac{\pi}{30}$ rad s^{-1}.

3. Work done by centripetal force will be zero. Banking angle depends on the velocity of the body and on the radius of the curvature of the road.

4. 1 rps = 2 π rad. $\frac{mv^2}{r}$ is the equation of centripetal force. Angular velocity of hand of the second > angular velocity of hand of the minute > angular velocity of hand of hour.

5. Unit of angular momentum is kgm^{2}s^{-1} and dimension is ML^{2}T^{-1}. $F = \frac{mv^2}{r} = m\omega^2 r$, $L = mvr = mr^2\omega$.

6. Another name of torque is rotational force. Dimension of angular momentum is ML^{2}T^{-1}. Unit of torque is N-m or Joule.

7. Radius of gyration of a thin circular disc is $K = \frac{r}{\sqrt{2}}$. Moment of inertia of a body depends on angular velocity. Moment of inertia of a thin uniform bar rotating through one end and through the centre is 4 times.
8. Moment of inertia of a body depends on mass and on the position of rotation axis. Unit of torque is N-m and dimension $[ML^2T^{-2}]$.
9. If the collision between two bodies of equal masses is elastic and initial velocity of the first body is u_1, final velocity v_1 and initial velocity of the second body is u_2 and final velocity v_2 then $u_1 = v_2$. The weakest force is gravitational force. Strong nuclear force is a strong force.
10. Unit force will be produced when unit acceleration is generated on a unit mass. Unit of angular momentum is kg m^2s^{-1}.
11. If the radius of a body rotating with uniform velocity is double then torque will be 4 times. Dimension of angular momentum = $[ML^2T^{-1}]$.
12. Angle between linear velocity and angular velocity is 90°. If a fan rotates 60 times per minute then its angular velocity will be 2π rad s^{-1}. Frequency of a minute-hand is 2.78×10^{-4} Hz.
13. While towing a boat vertical component of applied force is balanced by the helm of the boat.
14. Banking is the source of providing centripetal force in the curvature of a road. Angle between action and reaction is 180°.
15. For one complete rotation in a circular path of radius r displacement becomes $2\pi r$. Dimension of angular acceleration $[T^{-2}]$.
16. Impulse of force is equal to the rate of change of momentum. And impulsive force is the very large applied force acting for very small duration.
17. Moment of force or torque (i) $\vec{\tau} = \vec{r} \times \vec{F}$ (ii) $\vec{\tau} = I\vec{\alpha}$ (iii) $\vec{\tau} = \frac{d\vec{L}}{dt}$ (iv) $\vec{L} = \vec{r} \times \vec{P}$ (v) $E = \frac{1}{2} I\omega^2$ ($E \propto I$ when ω is constant), vector form of centripetal force : $-m(\vec{\omega} \cdot \vec{\omega})\, r$. Angle between action and reaction is 180°.
18. Action and reaction force (i) act on two bodies, (ii) their summation is zero.
19. For rotational motion (i) work = torque × angular velocity, (ii) power = torque × angular velocity.
20. Moment of inertia of two bodies depends on (i) the position of rotational axis (ii) shape of two rigid bodies (ii) rigid body around the axis of rotation. Force × time of action = impulse force.
21. If the mass of the first body is very large compared to the second body, then after collision the first body will continue to travel with the same velocity.

22. Angular velocity of hour-hand = $\frac{\pi}{21600}$ rad s^{-1}. Relation between moment of inertia and rotational kinetic energy, E = $\frac{1}{2}$ Iω^2. The ratio of kinetic energy of a body rotating with uniform velocity and the moment of inertia is proportional to the square of angular velocity. Moment of inertia of a body rotating with unit angular velocity is twice the kinetic energy.
23. If momentum is increased by 100%, then kinetic energy will change by 300%.
24. Moment of inertia of a rotating body with unit angular velocity is numerically equal to the double of its kinetic energy.
25. Radius of gyration of a rigid body is, K = $\sqrt{\frac{l}{M}}$. If the applied torque on a particle is zero, then angular momentum will be constant. While jumping in a diving angular momentum of the swimmer remains constant. Strongest force is strong nuclear force.
26. Product of the radius vector of a rotating particle and linear momentum with respect to a point is called angular momentum. Vector form of centripetal force is $\frac{m(\vec{v} \times \vec{v})}{r}$.
27. Relation between linear velocity v of a body rotating in a circular path and time period T is, $v = \frac{2\pi r}{T}$.
28. In elastic collision kinetic energy and momentum remain conserved. In inelastic collision total energy does not remain conserved. Angle betwen $\vec{r}$ and $\vec{P}$ is 90° for a particle rotating in circular path with respect to the centre. Banking angle depends on the velocity of the body and on the radius of curvature of the path.
29. In collision between two bodies action and reaction forces are—(1) equal and opposite, (2) always act on the same body.
30. Unit of impulse of force is Newton-second, relation between momentum and kinetic energy is, $E_k = \frac{p^2}{2m}$
31. Strong nuclear force is attractive, short range and charge neutral. Gravitational force does not depend on the nature of medium. Intensity of gravitional force is 1. Intensity of strong nuclear force is 10^{42}. The weakest force is the gravitational force.
32. Relation between kinetic energy and moment of inertia, $E_k = \frac{1}{2}$ Iω^2. Moment of inertia of a disc of mass M and radius R rotating with respect to the normal axis is $\frac{MR^2}{2}$. Moment of inertia of body depends on mass and axis of rotation. Linear velocity of particles near the ends of the wheel of the crusher of lentils is more and linear momentum of each particle is same.
33. Electromagnetic force is responsible for atomic structure. Angular momentum for a body for circular motion is $mr^2\omega$.