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TEXT BOOK

Physics

1st Paper

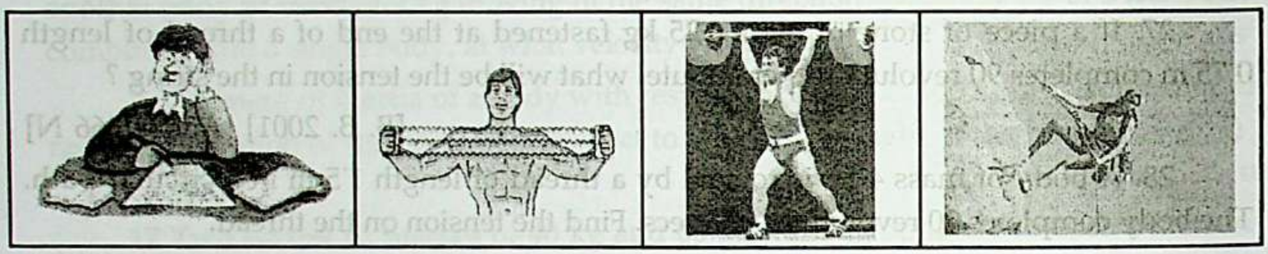
UNMESH

Medical & Dental Admission Care

5

WORK, ENERGY AND POWER

Key Words : Work, unit of work, energy, elastic force, kinetic energy, potential energy, power, unit of power, non-conservative force, efficiency.



Introduction

The three words **work, energy and power** are very familiar to us. In our daily life we use the word work for physical or mental work. So, in general term, **to do something is work.** For example, when a rickshawpuller pulls a rickshaw he does work, when a porter carries luggage he works, when a horse pulls a carriage then it works etc. So it is clear from it that in our daily life the word work is used in general form, not in any particular form. In physics work has a particular meaning. In general, we use power and energy for the same purpose. But actually these are not same. In this chapter we will give real explanation of work, energy and power and relation among them will be discussed.

After studying this chapter students will be able to—

- explain universal concept of work and energy
- analyse vector relation of work with force and displacement
- analyse work done by the constant and variable forces
- compare work done against elastic force and gravitational force
- derive mathematical equation of kinetic energy and will be able to use this equation for solving problems

Practical : Determination of potential energy of a spring

- solve different problems using conservation principle of energy
- analyse relation between power, force and velocity
- explain conservative and non-conservative forces
- calculate efficiency for a system

5·1 Universal concept of work and energy

5·1·1 Work

In general, to do something is called work. For example— to study, to work in factories, cycling etc. But in the language of science work has different meaning.

If there is displacement of a body due to application of force only then work is done. For example, a book is allowed to fall from the table to ground. A person reached to the top of a hill step by step by carrying a load on his head. By these two examples, we can explain work. In the first case, displacement has been done along the direction of gravitational force and in the second case, displacement has been done against the gravitational force. So, work has been done in both the cases. A person standing at a fixed place carrying a load on his shoulder has become very tired, but no work is done in this case as there is no displacement. From this discussion **it is understood that if there is displacement due to application of force only then work will be done. But if there is no displacement even if force is applied, no work will be done.**

If there is displacement of s due to the application of force F, then work

$$W = Fs \qquad \ldots \qquad \ldots \qquad \ldots \qquad (5.1)$$

Work is a scalar quantity. In vector form it can be written as,

$$W = \vec{F} \cdot \vec{s} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (5.2)$$

In our daily life we see many examples of work around us. Boys are playing football and rickshawpuller is pulling rickshaw, vendors are selling different products, the cultivator is ploughing in the field etc.

Let us look at the following examples :

(1) Milon is standing in the class with books.

(2) Rana is pushing a wall with both of his hands.

(3) Rimi sent a toy from one end to the other end of the room by pushing.

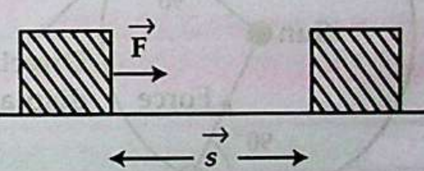


Fig. 5·1

Since work will be done only when there will be a motion of a body due to application of force, hence in the first and second cases no work has been done; but in the third case work has been done.

Again if angle θ is produced along the direction of s due to the application of force F, then work can be expresed by the product of force and the component of displacement. That means, work = force × component of displacement along the direction of force

$$\text{or, } W = Fs \cos \theta \qquad \ldots \qquad \ldots \qquad \ldots \qquad (5.3)$$

So work can be defined in the following way.

Definition : If force is applied on a body and if there is a displacement of the body, then product of the force and the component of displacement along the direction of force is called work.

Unit : S. I. unit of work is Joule or Newton-metre (N-m). Work is a scalar quantity.

Amount of work done for the displacement of 1 metre of a body due to the application of force of 1 Newton is called 1 Newton-metre or 1 Joule.

Gravitational unit of work : kg-metre

Demension of work : $[W] = [F][s] = MLT^{-2} \times L = ML^2T^{-2}$

Perceptional work : Why work is a scalar quantity even though force and displacement are vector quantity.

Work is the dot product of force and displacement i.e., $W = \vec{F} \cdot \vec{s} = Fs \cos\theta$. Since dot product is a scalar quantity so force and displacement being vector quantity, work is a scalar quantity.

5·1·2 Read the following events and learn/know the causes for work done and work not done

● Earth is rotating round the sun, the direction of displacement of the earth at any time is along the tangent of the arc [Fig. 5·2]. But the gravitational force by which the sun attracts the earth always points towards the sun from the earth; that means, acts along the radius of the arc towards the centre. So, the direction of the attractive force of the sun and the displacement of the earth are mutually perpendicular to each other, hence during rotation of the earth no work is done by the gravitational force of the sun.

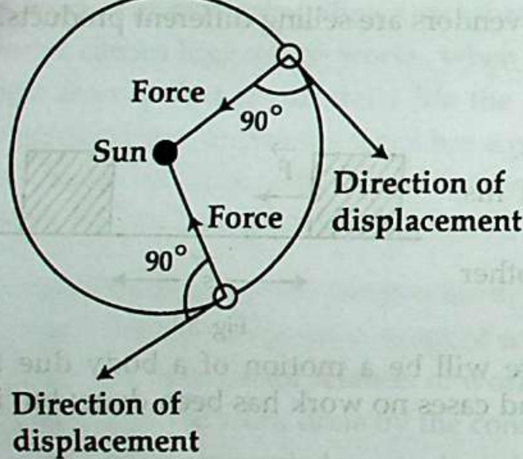


Fig. 5·2

● Walking along a plane road carrying a bag in hand, the weight of the bag i.e., gravitational force does not do any work. This is because, that while walking along the road, displacement of the bag is along the horizontal line, i.e., along the perpendicular direction of the gravitational force. So, although there is displacement of the bag but gravitational force does not do any work. Hence in this case gravitational force is workless. But while walking in uneven or undulated road work is done by the gravitational force.

● If a stone is made to rotate by holding it by hand through a rope, then the stone moves in a circular path. Here the tension of the rope is centripetal force. So, during the rotation of the stone the tension of the rope does not do any work.

Work : Put on the ground a shrimp that has just been taken out of water. Now touch the shrimp from a distance by a stick. What will you see ? The shrimp will jump straight upward. Will the shrimp do any work in this case ?

From the definition of work we know that though force acts,

(i) if displacement of the point of action of the applied force $s = 0$, then work, $W = 0$.

(ii) if the displacement of the point of action of the applied force is perpendicular to the force i.e., $\theta = 90°$, then $\cos\theta = \cos 90° = 0$, so $W = 0$. So, no work is done in this case.

5·1·3 Work done in some special cases

Work done by more than one force

If more than one force is applied on a body then total work done by those forces will be the sum of work done by individual force. Wok done by the resultant force is also same.

Work done by the force

If a force is applied on a moving football along its direction of motion, then the football moves towards the direction of the force. When a mango falls on the ground from a tree, due to the action of gravitational force it falls directly downward. In both the cases, work is positive or it is understood that work is done by the force. So, it can be said that when the point of action of the applied force moves along the direction of force or if there is a component of the displacement along the direction of the force, then it is understood that work is done by the force. In this case work done by the force is **positive work.** If work is done along the direction of force then potential energy decreases. For work done by the force $0° \leq \theta \leq 90°$.

Work done against the force

A person has lifted a bag of rice on his head from the ground. Again, a book has been lifted in the almirah from the floor or a table. In these two cases, work is done against the gravitational force. So, if a body is moved against the point of action of the applied force i.e., the point of action of the force moves against the direction of the force, then it is understood that work is done against the force. In this case work done against the force is **negative work.** If work is done against the force then potential energy increases. For work done against force $180° \geq \theta \geq 90°$.

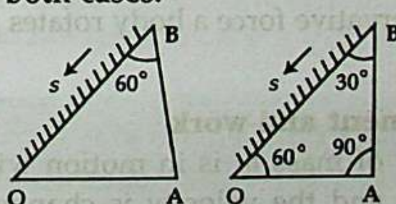
Zero work : Due to the application of force on a body if the displacement is in the perpendicular direction of the force then work done by that force is zero or no work is done. In that case $\theta = 90°$ and work $W = Fs \cos 90° = 0$.

Perceptional work : The earth is rotating around the sun but why no work is done ?

The earth is rotating around the sun, but no work is done. This is because that at each moment displacement of the earth takes places along the normal of the gravitational force. That means, $W = Fs \cos \theta = Fs \cos 90° = 0$. Hence no work is done.

Mathematical examples

1. A person of mass 150 kg came down along a stair of 4m long with a load of 50 kg. If the inclination of the stair is 60° with the wall and 60° with horizontal, find the work done by the person in both cases.



Here,
Mass, $m = 150 + 50 = 200$ kg
Acceleration due to gravity, $g = 9{\cdot}8\ ms^{-2}$
Angle, $\theta = 60°$
Work done, $W = ?$

In the first case :
We know,

$W = Fs \cos \theta$
$= mg\, s \cos \theta$

$\therefore\ W = 200 \times 9{\cdot}8 \times 4 \times \cos 60°$
$= 3920$ J

In the second case :
Again in case of 60° with the horizontal, $\theta = 90° - 60° = 30°$

$\therefore$ work, $W = Fs \cos 30° = 200 \times 4 \times 9{\cdot}8 \cos 30° = 6789{\cdot}4$ J

2. The displacement of a particle is $\vec{r} = (3\hat{i} - 2\hat{j} + \hat{k})$ m when force $\vec{F} = (5\hat{i} + 3\hat{j} - 2\hat{k})$ N is applied on it. What is the work done by the force?

We know,

$$W = \vec{F}.\vec{s}$$

$$\therefore W = (5\hat{i} + 3\hat{j} - 2\hat{k}).(3\hat{i} - 2\hat{j} + \hat{k})$$

$$= (5 \times 3 - 3 \times 2 - 2 \times 1) \text{ J}$$

$$= (15 - 6 - 2) \text{ J} = 7 \text{ J}$$

Here,

Force, $\vec{F} = (5\hat{i} + 3\hat{j} - 2\hat{k})$N

Displacement, $\vec{s} = (3\hat{i} - 2\hat{j} + \hat{k})$ m

Work done, W = ?

3. An object of mass 5 kg falls from a height of 5 m on a nail and the nail enters 10 cm inside the ground. Calculate the average resisting force of the ground.

[C. B. 2006]

We know,

Potential energy of the falling body = Work done against the resisting force

Now, work done against the resisting force,

$$W = F \times s$$

$$\therefore W = F \times 0{\cdot}1 \quad \dots \quad \dots \quad \dots \quad \text{(i)}$$

Total fall of the object = $h + s = 5 + 0{\cdot}1 = 5{\cdot}1$ m

$\therefore$ Potential energy of the object $= mg\,(h + s) = 5 \times 9{\cdot}8 \times 5{\cdot}1$

Here,

mass, $m = 5$ kg

height, $h = 5$ m

displacement, $s = 10$ cm $= 0{\cdot}1$ m

resisting force, F = ?

Now according to question,

$$F \times 0{\cdot}1 = 5 \times 9{\cdot}8 \times 5{\cdot}1$$

$$\therefore F = \frac{5 \times 9{\cdot}8 \times 5{\cdot}1}{0{\cdot}1} = 2499 \text{ N (Ans.)}$$

Perceptional work : Under what conditions work is zero.

Conditions for zero work :

(a) If displacement is zero, then work, $W = F.s = F \times 0 = 0$.

(b) If angle between force and displacement is 90°, then work, $W = Fs \cos 90° = 0$.

(c) If due to conservative force a body rotates in a circular path then work will be zero.

5·2 Force, displacement and work

Suppose a marble of mass m is in motion with initial velocity of v_0. A force is applied on this marble and the velocity is changed to v. So, kinetic energy before applying force $= \frac{1}{2}mv_0^2$ and the kinetic energy after applying force $= \frac{1}{2}mv^2$. Here, the work done in this case will be equal to the difference of the two kinetic energies.

$\therefore$ Work = Change of kinetic energy

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \quad \dots \quad \dots \quad \dots \quad (5.4)$$

From the equation of motion we know,

$$v^2 = v_0^2 + 2as \quad \ldots \quad \ldots \quad \ldots \quad (5.5)$$

Here a = acceleration, s = displacement. Now by multiplying both sides of equation (5.5) by $\frac{1}{2}m$ we get,

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}m(2as)$$

$$\text{or, } \frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + mas \quad \ldots \quad \ldots \quad \ldots \quad (5.6)$$

$$\text{or, } \frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + Fs \quad \ldots \quad \ldots \quad \ldots \quad (5.7)$$

$$[\because F = ma]$$

$$\text{or, } \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = Fs \quad \ldots \quad \ldots \quad (5.8)$$

From equations (5.4) and (5.8) we get,

$$W = Fs \quad \ldots \quad \ldots \quad \ldots \quad (5.9)$$

If component of displacement along the direction of force is considered instead of considering applied force along displacement, then according to fig. 5.3

$$W = Fs \cos\theta$$

If expressed in vector form, we get

$$W = \vec{F} \cdot \vec{s} \quad \ldots \quad \ldots \quad (5.10)$$

So we can say that scalar product of force and displacement is work done. That is, product of displacement and component of force along displacement is work done.

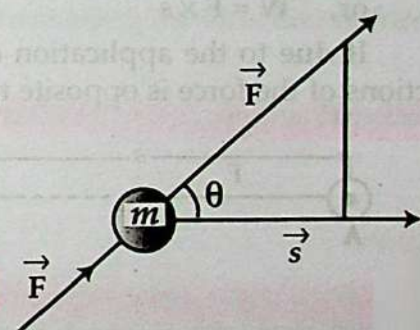


Fig. 5.3

From general definition of work it is seen that even if force acts but displacement of the point of action of force $s = 0$, then work done $W = 0$. Again, if displacement of the point of action of the force is perpendicular i.e., $\theta = 90°$, then $W = Fs \cos 90° = 0$.

That means, one or more forces can act on a body perpendicular to the direction of motion. If point of action of these forces makes an angle of 90° with the direction of displacement, then these forces do not perform any work. This type of force is called **workless force.**

Preceptional work : If a body revolves or rotates with uniform speed will there be any work done ? Explain.

No work is done for a body revolving with uniform speed. In this case applied tension or force applied by hand on a body acts as centripetal force. At every small moment angle between the applied force $\vec{F}$ and associated small displacement ($d\vec{s}$) is 90°. As $\vec{F}$ is along the centre of the circle and $d\vec{s}$ is along the tangent of the circle, hence work, $W = Fs \cos\theta = Fs \cos 90° = 0$.

5·3 Work done by constant force and variable force

Generally there are two kinds of force, viz.—constant force and variable force. Now we shall discuss constant force and variable force in detail.

5·3·1 Work done by a constant force

A body can be raised above or lowered down by small amount in the sphere of gravitational force. Since the height is small, so we can consider gravitational force constant, ($\because$ F = mg, as the height is small, magnitude of g is considered constant, so F is constant.)

That means, if the magnitude and direction of the force is not changed with respect to time, then that force is called constant force.

Let a force F be applied at point A of a body along AB and due to this the body moving from point A to point B covers a distance s [Fig. 5·4(a)]. Then,

work done = magnitude of the force × magnitude of displacement along the point of action of the force

or, $W = F \times s$ ... ... (5.11)

If due to the application of force the displacement of the body i.e., the point of actions of the force is opposite to the direction of force i.e., AB = s [Fig. 5·4(b)], then

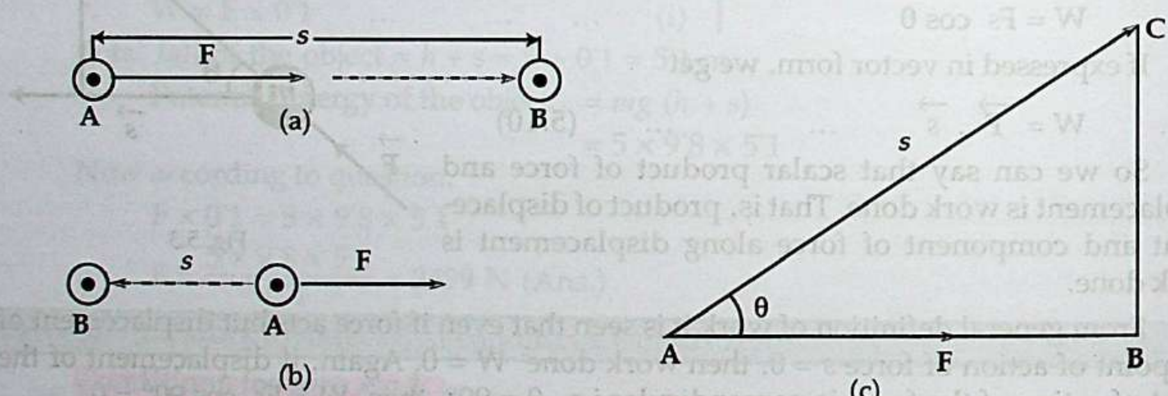


Fig. 5·4

work done = magnitude of the force × magnitude of displacement along the direction of force,

i.e., $W = F \times (-s) = -F \times s$ ... ... (5.12)

Negative sign is used to indicate that force and displacement are opposite to each other. If you push a snake with a stick and the snake rushes towards you, then work done is negative and $W = -Fs$.

Now, consider that due to the action of force F on a body along direction AB, the body reaches to point C covering a distance s making an angle θ with the direction of the applied force [Fig. 5·4(c)]. Then displacement of the body along the line of action of the force = AB = $s \cos\theta$.

Here, BC ⊥ AB

∴ Work done, W = magnitude of force × magnitude of displacement along the direction of force

or, **W = Fs cos θ** ... ... (5.13)

= magnitude of force × component of displacement along the line of action of the force

or, W = Fs cos θ = magnitude of displacement × component of force along the direction of force.

In both cases the amount or volume of work is same.

By vector algebra work can be expressed as :

Work is measured by the scalar product of two vectors, force and displacement.

Suppose, force $\vec{F}$ is a vector quantity and displacement $\vec{s}$ is also a vector quantity.

So, work = force . displacement

or, $\mathbf{W} = \vec{F} . \vec{s}$

$= \vec{s} . \vec{F} = Fs \cos\theta$, ... (5.14)

[$s \cos\theta$ is the component of displacement along the direction of force F]

Here, θ = angle between $\vec{F}$ and $\vec{s}$.

(a) Positive work : If θ = 0°, i.e., when there is displacement along the direction of force, then

$$W = \vec{F} . \vec{s} = Fs \cos\theta = Fs \cos 0°$$
$$= Fs \qquad [\because \cos 0° = 1]$$

Here work is positive. In other words, if θ is an acute angle, then work will be positive. **If work is positive, then it means that work is done by the force. In case of positive work kinetic energy increases, acceleration occurs.**

(b) Zero work : If θ = 90°, then

$$W = F.s \cos\theta = F.s \cos 90° = 0 \qquad [\because \cos 90° = 0]$$

That means, if θ = 90°, then work done will be zero.

Work one by centripetal force is zero. The direction of centripetal force is along the radius and towards the centre and direction of displacment is along the tangent of the circle. So, θ = 90° and work is zero.

(c) Negative work : If θ = 180°, then work done will be negative.

i.e., $W = \vec{F} . \vec{s} = Fs \cos 180° = -Fs \qquad [\because \cos 180° = -1]$

If work is negative, then it means that work is done against the force. For negative work kinetic energy decreases and retardation occurs.

Perceptional work : Work done by a rotating body in circular path is zero—expalin.

When a body is rotated along a circular path, then at every moment angle between the centripetal force (F) and small displacement (s) is, θ = 90°. So, $W = \vec{F} . \vec{s} = Fs \cos\theta = 0$. That means, as the component of displacement along the centripetal force is zero, hence no work will be done.

5.3.2 Work done by a variable force

Definition : If magnitude and direction of a force or any one of them is changed, then that force is called a variable force i.e., changeable force. For example, work done in stretching or contracting a spring is called work done by a variable force. Again, change of position of a body in a gravitation field also means work done by a variable force.

For small height, the change of force is exceedingly small. But gravitational force for large height and depth from the surface of the earth continues to decrease. In that case force cannot be considered to be constant. Force is a vector quantity; so it has both magnitude and direction. Firstly, considering force to be variable, we will derive below the equation for work done.

A. When the magnitude of force is varying or changeable

Let a varying force $\vec{F}$ be acting on an object along x-axis. The object moves away from x_1 to x_2 due to the action of the force on the object. The direction of $\vec{F}$ is along x-axis, so it is fixed but the magnitude of it is varying.

Now, we will find the work done by this varying force for the displacement $(x_2 - x_1)$ of the object.

In order to calculate the work done let the entire displacement $(x_2 - x_1)$ be divided into a large equal number of small segments [Fig. 5.5(a)]. Since the intervals are very small the force along each of them may be taken as constant. Let F_1, F_2, F_3 .... F_n be the magnitudes of the forces over the intervals $\Delta x_1, \Delta x_2, \Delta x_3$ .... Δx_n. But since we have considered that all the intervals are small, so $\Delta x_1 = \Delta x_2 =$ .... $\Delta x_n = \Delta x$, but the force acting on each interval is different. So the work done for the displacement between x_1 to $x_1 + \Delta x$ is given by,

$$\Delta W_1 = F_1 \Delta x$$

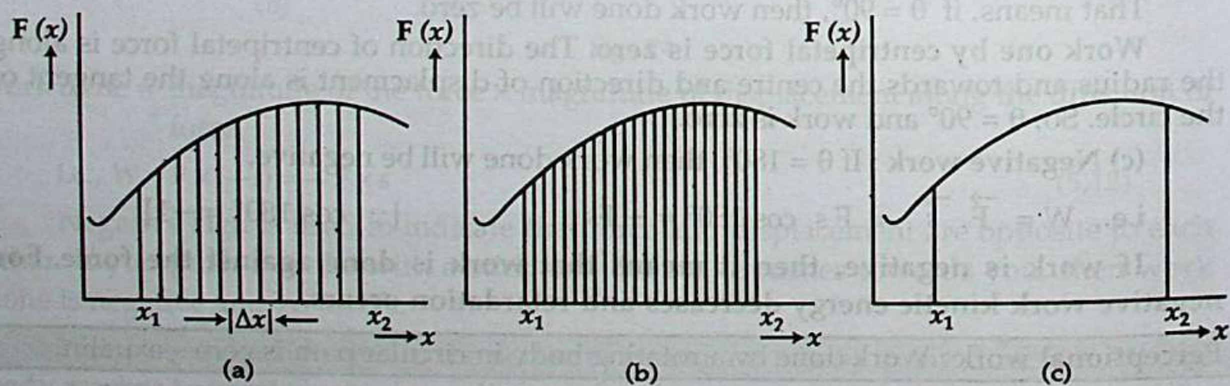


Fig. 5.5

Similarly, from $x_1 + \Delta x$ to $x_1 + 2\Delta x$ the displacement is Δx, and the force is F_2, hence work done, $\Delta W_2 = F_2 \Delta x$

If total displacement $(x_2 - x_1)$ is divided into N number of similar small displacement Δx, then total work done will be equal to the summation of those small elements of displacement,

So, work done,

$$W = \Delta W_1 + \Delta W_2 + \Delta W_3 + \ldots\ldots\ldots + \Delta W_n$$
$$= F_1\Delta x + F_2\Delta x + F_3\Delta x + \ldots\ldots\ldots + F_n\Delta x$$
$$= \sum_{k=1}^{N} F_k \Delta x$$

We have considered that for a particular interval the force is constant, which is not exactly true. If we make the division further smaller and smaller so that $\Delta x \to 0$, then the assumption of constant force in each interval will be more accurate [Fig. 5·5(b)]. Hence total work done is given by,

$$W = \underset{\Delta x \to 0}{\text{Lt}} \sum_{k=1}^{N} F_k \Delta x$$

The process of summation becomes equivalent to integration and the work done can be expressed as,

$$\underset{\Delta x \to 0}{\text{Lt}} \sum_{k=1}^{N} F_k \Delta x = \int_{x_1}^{x_2} F(x)\, dx$$

$$\therefore\ W = \int_{x_1}^{x_2} F(x)\, dx \quad \ldots \quad \ldots \quad \ldots \quad (5.15)$$

= area of the graph within limits x_1 and x_2 [Fig. 5·5(c)]

If the angle between the force and the displacement is θ [Fig. 5·6], then

$$W = \int \vec{F} \cdot d\vec{x} = \int_{x_1}^{x_2} F\cos\theta\, dx, \quad \ldots \quad \ldots \quad \ldots \quad (5.16)$$

Here $F\cos\theta$ is the component of $\vec{F}$ along X-axis.

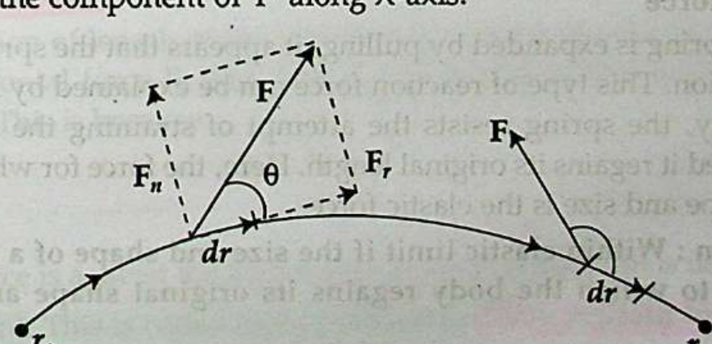


Fig. 5·6

B. When both magnitude and direction of the force are varying or changeable

When the magnitude and direction of the force acting on an object are varying, the object can move in a line and the motion can be two or three dimensional. In this case, tangent drawn at a particular point of the line gives the direction of the object at that point. Here displacement = $\vec{r}$.

In order to determine the work done for this type of force, the entire path of motion can be considered as the summation of extremely small segment of displacement $d\vec{r}$.

For each small segment of displacement the force at the initial position and final position can be considered to be constant.

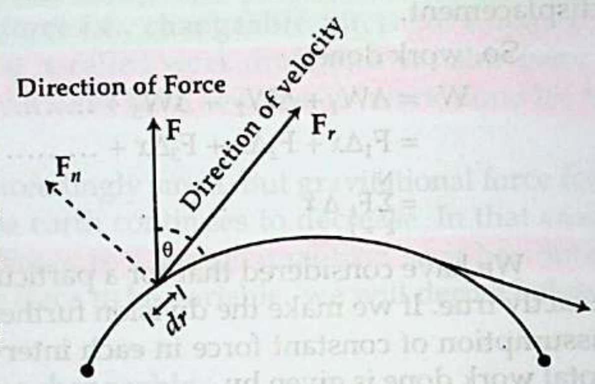


Fig. 5'7

Let the angle between a segment of displacement $d\vec{r}$ and the force acting for that displacement is θ [Fig. 5'7]. Now let the force be resolved into two components. One is parallel to $d\vec{r}$ and the other one is perpendicular to $d\vec{r}$. Let the components be $F_r = F\cos\theta$ and $F_n = F\sin\theta$ respectively.

But work done by the force F_n is zero [$\because \vec{F}.d\vec{r} = F\cos 90°.dr = 0$]. So the work done by the force $\vec{F}$ for the displacement $d\vec{r}$ is,

$$dW = F\cos\theta\, dr = \vec{F}.d\vec{r}$$

So, the total work done for the displacement between the position r_0 to position r is,

$$W = \int_{r_0}^{r} (F\cos\theta)\,dr = \int_{r_0}^{r} \vec{F}.d\vec{r} \quad \ldots \quad \ldots \quad \ldots \quad (5.17)$$

5'4 Elastic force and gravitational force and work done

5'4'1 Elastic force

When a spring is expanded by pulling, it appears that the spring pulls our hands in opposite direction. This type of reaction force can be explained by Newton's third law of motion. Clearly, the spring resists the attempt of straining the spring and when the spring is released it regains its original length. Here, the force for which the spring regains its original shape and size is the elastic force.

Definition : Within elastic limit if the size and shape of a body is changed and the force due to which the body regains its original shape and size is called the elastic force.

If due to application of force a displacement x is produced in the spring then work done by the spring, $W = \frac{1}{2}kx^2$. Here k **is spring constant or force constant.** Again work done will be same if same amount of force is applied to contract the spring by displacement x. That means, in both cases work will be same. **So, work done by elastic** force is proportional to the square of displacement i.e., $W \propto x^2$. If dispalcement is double against the elastic force, then work will be four times.

5·4·2 Work done by gravitational force

If a body is brought from top to bottom or is taken from bottom to top, then work is done by gravitational force. That means, whatever is done, either to raise the body or bring it down, the body is always being attracted towards the centre of the earth by a force. This attractive force of the earth is called gravitation.

If the radius of the earth is R and mass is M then work done to raise or lower a body of mass m to a height h will be, $W = \frac{GMm}{R^2} \times h$, here $\frac{GMm}{R^2}$ is constant. **So, work done by gravitational force is proportional to the height or displacement. That means, $W \propto h$.** Hence if displacement against gravitational force is three times, then work done will also be three times.

5·4·3 Examples of work done by variable force

A. Work done in expanding a spring ($F \propto x$) or work against elastic force i.e., against force of spring

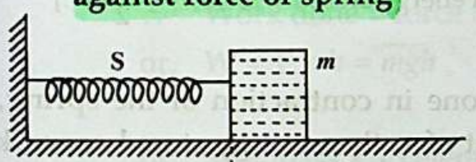


Fig. 5·8(a)

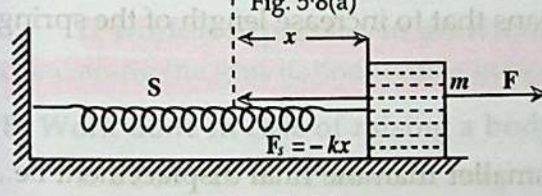


Fig. 5·8(b)

Let one end of a horizontal ideal spring S be fastened with a wall and an object of mass m be attached at the other end. The body can move on the horizontal and frictionless plane [Fig. 5·8(a)].

Now, if by pulling the object the spring S is strained along its length, then due to elastic property a force will develop in the spring which will be same and will act against the applied force. This is called **restoring force.** According to Hooke's law, within elastic limit, the magnitude of restoring force will be proportional to the applied force.

Let x be the extension of length of the spring along the horizontal direction due to the application of horizontal force F_s [Fig. 5·8(b)]. Due to this action, a restoring force $-kx$ will be developed. This is because,

$F_s \propto -x.$

or, $F_s = -kx$

[Since restoring force is against the displacement, so negative sign is used]

Here, k is a constant. This is called **spring constant. Force applied to increase the length of the spring by unity is called force constant of that spring.**

In order to expand the spring, equal amount of external force is to be applied in the spring. Let the applied force be F.

$\therefore \quad F = -F_s = -(-kx) = Fx \quad \ldots \quad \ldots \quad \ldots \quad (5.18)$

In expanding the spring from position x_1 to position x_2, the amount of work done is;

$$W = \int_{x_1}^{x_2} \vec{F}(x) \cdot \vec{dx} = \int_{x_1}^{x_2} F(x)\, dx \qquad [\because \text{angle between } \vec{F} \text{ and } \vec{dx} \text{ is zero}]$$

$$= \int_{x_1}^{x_2} kx dx = k\int_{x_1}^{x_2} x dx = \frac{1}{2}k\,[x^2]_{x_1}^{x_2} = \frac{1}{2}k\,[x_2^2 - x_1^2]$$

$$\therefore \quad W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 \qquad \dots \qquad \dots \qquad \dots \qquad (5.19)$$

This work is positive, which remains stored as potential energy in the spring.

If the initial position of the spring, $x_1 = 0$ and final position, $x_2 = x$, then from equation (5.19) we get,

$$W = \frac{1}{2}kx^2 \qquad \dots \qquad \dots \qquad \dots \qquad (5.20)$$

That is, if displacement is x, stored potential energy is $\frac{1}{2}kx^2$.

[N. B. If the spring is contracted by an amount x, then energy stored will be $W = \frac{1}{2}kx^2$.]

B. In case of contraction of the spring

In this case, if $x_1 = 0$ and $x_2 = x$, then work done in contraction of the spring, $W = \frac{1}{2}kx^2$ i.e., if the spring is contracted by an amount of x, then energy stored or work $= \frac{1}{2}kx^2$. Spring constant of a spring is 2·5 Nm^{-1} means that to increase length of the spring by 1 m, 2·5 N force is to be applied.

C. Negative work done by spring force

If the magnitude of initial displacement is smaller than the final displacement i.e., $|x_1| < |x_2|$, then the spring will do negative work on the body. Here, $F = F_s = -kx$ and if $x_1 = 0$ and $x_2 = x$, then work, $W = -\frac{1}{2}kx^2$.

5·4·4 Force due to gravity

An attractive force acts between any two bodies in this universe. Generally attraction between two bodies is called gravitation. **But the attractive force between the earth and a body above or near the surface of the earth is called force due to gravity.** So, gravity is the special case of gravitation, i.e., gravity means gravitation of the earth.

If a body is allowed to fall, the body falls straight downward due to gravity. The body also attracts the earth with equal and opposite force. Now mass of the earth is many times heavier than any body on the earth, hence any motion of the earth due to the attractive force of the body can be ignored. So, the body falls towards the earth, the earth does not move towards the body.

If the earth is considered as a homogeneous sphere of radius R we can consider that all its mass is concentrated at the centre of the earth. So, if the earth attracts a body on the surface of the earth of mass m towards its centre with a force F, then according to Newton's law of gravitation,

$$F = \frac{GMm}{R^2} \quad \ldots \quad \ldots \quad \ldots \quad (5.21)$$

This very force towards the centre of the earth is the force due to gravity. So, a falling body actually approaches towards the centre of the earth due to gravity. For this reason the suspended needle or plummet of the mason always points towards the centre of the earth. To bring down a body through pulley, to raise a body upward by a crane and to slip through the polished surface by children in children park— all these are work done by gravity.

5·4·5 Examples of work done by gravitational force

A. Work done in case of falling of a body

Suppose, a body of mass 'm' has been thrown from height 'h' due to action of gravity.

$\therefore$ Work done = force × displacement

or, $W = F \times h = mgh$ ... ... (5.22)

$[\because F = mg]$

or, W = mass × acceleration due to gravity × height

If work is expressed in gravitational unit, $W = mgh$. That means $W \propto h$. So work done along the gravitational force is proportional to displacement or height.

B. Work done in case of raising a body

If a body of 'm' is raised at height 'h' against the force of gravity, then

Work = mass × acceleration due to gravity × height

This work is, of course, negative.

i.e., $W = -mgh$ ... ... (5.23)

That means, $W \propto h$ i.e., work done against the gravitation is proportional to the height or displacement of the body.

C. Work done in bringing down through an inclined plane

Let a body of mass 'm' move from point A to B through a polished inclined plane. If g is acceleration due to gravity, then force of gravity will pull the body directly downward [Fig. 5·9].

Suppose the angle between displacement and the direction of the force of gravity is θ and AB = s.

$\therefore$ Component of displacement along the force of gravity $mg = s\cos\theta$

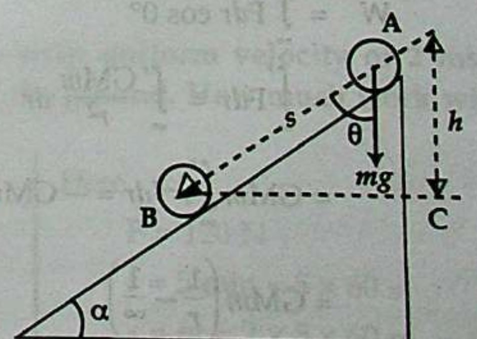


Fig. 5·9

If there were no inclination, the time at which the body moves from A to B, it would have come down by AC = h distance.

$\therefore\ h = s\cos\theta$

$\therefore$ Work, $W = mgs\cos\theta$ or, $W = mgh$ ... ... (5.24)

If the surface remains at an angle α with the horizontal, then $\theta = (90° - \alpha)$

$\therefore\ W = mgs\cos(90° - \alpha) = mgs\sin\alpha$

It is seen from work done by elastic force and force of gravity that,

work done by elastic force, $W = = \frac{1}{2}kx^2$ $\therefore$ Work, $W \propto$ (displacement)^2

On the other hand, work done by the force of gravity,

$W = mgh$ $\therefore$ Work, $W \propto$ displacement.

So, it can be said that **work done by elastic force is directly proportional to the square of displacement. On the other hand work done by force of gravity is directly proportional to displacement or height. If the magnitude of acceleration due to gravity is increased work done by this force also increases.**

D. Work done by gravitational force

Suppose a body of mass M is located at point P in the gravitational field. At a distance r from P another body of mass m is located at point Q [Fig 5·10]. In this case, gravitational force acting on the body of mass m is, $F = \frac{GMm}{r^2}$, the direction is along QP.

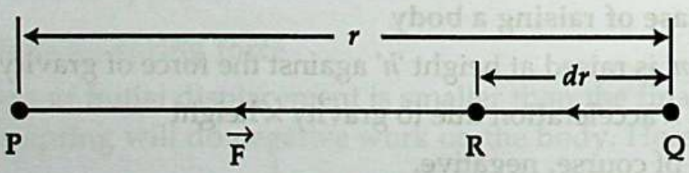


Fig. 5·10

Now, work done in bringing the body of mass m from infinity to point R by displacing a small distance dr, is

$$W = \int_{\infty}^{r} F dr \cos 0°$$

$$= \int_{\infty}^{r} F dr = \int_{\infty}^{r} \frac{GMm}{r^2} dr$$

$$= GMm \int_{\infty}^{r} r^{-2} dr = -GMm\left[\frac{1}{r}\right]_{\infty}^{r}$$

$$= GMm\left(\frac{1}{r} - \frac{1}{\infty}\right)$$

$$= -GMm$$

Now by displacing the body of mass m from R to exceedingly small distance dr, work done by gravitational force to bring it to a point is,

$$dW = Fdr \cos 180° = -Fdr$$

If the initial position of the body is r_a and the final position of it is r_b, then to determine total work the above equation is to be integrated within limits $r = r_a$ to $r = r_b$ and we get

$$W_{ab} = \int_{r_a}^{r_b} -Fdr = -\int_{r_a}^{r_b} \frac{GMm}{r^2} dr$$

$$= -GMm\left[-\frac{1}{r}\right]_{r_a}^{r_b}$$

$$= -GMm\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

Positive work done by the gravitational force : According to the above equation if the distance between the two bodies is decreased, i.e., $r_b < r_a$, then $\frac{1}{r_b} > \frac{1}{r_a}$. So W is positive. Hence work done by gravitational force is positive. While falling from top distance decreases and work becomes positive.

Negative work done by the gravitational force : According to the above equation if $r_b > r_a$ i.e., if distance between two particles increases then $\frac{1}{r_b} < \frac{1}{r_a}$ in that case work is negative. When a body is lifted upward then distance increases, hence work done by the gravitational force becomes negative.

Work : What is the difference between work done by gravitational foce and elastic force?

Work done by gravitational force is proportional to distance i.e., $W \propto h$; on t
hand work done by elastic force is proportional to the square of distance i.e., W

Mathematical examples

1. A horse can drag and remove a body with uniform velocity of 2 ms^{-1} by applying force of 120 N at an angle of 30° with the ground. How much work will the horse do in 5 minutes?

Here,

$F = 120$ N

$t = 5 \text{ min} = 5 \times 60$ s

$s = vt = 2 \times 5 \times 60 \text{ m} = 600$ m

$\theta = 30°$

We know,

$$W = Fs\cos\theta$$

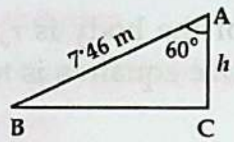
$$= 120 \times 600 \times 0{\cdot}866$$

$$= 6{\cdot}35 \times 10^4 \text{ J}$$

2. A ladder of length of 7·46 m is attached to a building at an angle of 60°. A person weighing 60 kg is ascending with a load of 15 kg to the roof through that ladder in 30 s. Calculate the applied power.

We know, work done against gravitational force,

W = weight, mg × vertical displacement, h



Here, $h = 7{\cdot}46 \text{ m} \cos 60°$

$m = (60 + 15)$ kg

$g = 9{\cdot}8 \text{ ms}^{-2}$

$t = 30$ s

∴ required work,

$$W = (60 + 15) \text{ kg} \times 9{\cdot}8 \text{ ms}^{-2} \times 7{\cdot}46 \text{ m} \cos 60°$$
$$= 75 \times 9{\cdot}8 \times 3{\cdot}73 \text{ J}$$

$$\therefore \text{ power, } P = \frac{W}{t} = \frac{75 \times 9{\cdot}8 \times 3{\cdot}73 \text{ J}}{30 \text{ s}}$$
$$= 91{\cdot}385 \text{ W}$$

3. A body in the horizontal plane is attached to a spring. The spring is contracted 3 cm from equilibrium by a force of 2·4 N. What will be the work done by the spring ?

We know,

$$\text{work, } W = \frac{1}{2}kx^2 = \frac{1}{2} \times 80 \times (0{\cdot}03)^2$$
$$= 3{\cdot}6 \times 10^{-2} \text{ J}$$

Here,

$F = 2{\cdot}4$ N

$x = 3 \text{ cm} = 0{\cdot}03$ m

$k = \frac{F}{x} = \frac{2{\cdot}4}{0{\cdot}03} = 80 \text{ Nm}^{-1}$

W = ?

Work done on a particle moving along a curved path

Suppose, a particle is moving in a curved path due to the action of variable force $\vec{F}$ [Fig. 5·11]. Total work done on the particle is W.

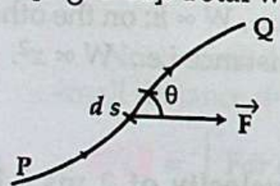


Fig. 5·11

According to the figure if the small displacement is $d\vec{s}$ then for travelling total path from P to Q total work done on the particle,

$$W = \int_P^Q \vec{F} . d\vec{s} = \int_P^Q Fs \cos\theta \, d\theta \quad \ldots \ldots \quad (5.25)$$

In equation (5.25) none of the quantity F or θ is constant.

5·5 Energy

When a body can work, it is considered that it has energy. Energy is measured by the total amount of work that a body can do. That means, **by work done** we can measure energy. When a body does work its energy decreases and the body on which work is done its energy increases. Energy has no mass, weight and volume. The body which has less capacity to do work has less energy. So, it can be said that **work is the**

measure of energy. If it is said that a body has done W amount of work, then it is understood that energy used by it is W. When a body does work against the force then it loses energy. Again, if work is done on a body by the force, then it gains energy.

Definition : Energy of a body is the capacity or ability to do work. It is measured by the amount of work the body can do. Like work energy is also a scalar quantity.

Amount of work = work done = applied force × displacement of point of action of the force.

Petrol vapour in a motor engine, water vapour pressure drives piston in a vapour engine. So, vapour has energy. Again, electricity has energy. Trains, factories and industries are run by electricity. The universe is moving because there is energy. Energy can be transformed, but can never be created nor destroyed. So, in transformation process total energy remains unchanged. We will learn in detail about it in conservation principle of energy. Energy has different forms, viz—

(i) Mechanical energy
(ii) Heat energy
(iii) Light energy
(iv) Sound energy
(v) Magnetic energy
(vi) Electrical energy
(vii) Chemical energy
(viii) Nuclear energy
(ix) Solar energy

In this chapter we will discuss mechanical energy.

5.6 Mechanical energy

Ability to work done or energy that exists in a body with respect to its environment or position for motion that energy is called mechanical energy.

Mechanical energy is of two types; viz.—
(1) Kinetic energy and
(2) Potential energy.

5.6.1 Transformation of energy

Energy is present in different forms in this universe. Different forms of energy are related to one another. It is possible to transform one form of energy to another form and it is called **transformation of energy.**

Some examples of transformation of energy are given below :

(1) Water flows from higher place to lower place. Its energy at higher place is potential energy. While flowing downward its potential energy changes to kinetic energy.

By rotating turbines by this kinetic energy electricity is generated. That means, **mechanical energy has been transformed into electric energy.**

(2) When electric energy passes through electric bulb, we get light. Here **electric energy has been transformed into light energy.**

(3) When current is passed through an electric iron, heat is produced. We iron clothes by this heat. Here **electric energy has been converted to thermal energy.**

(4) If an insulated cast iron is wrapped by copper wire and current is allowed to flow through it, then the iron plate becomes a magnet. Here **electric energy is converted into magnetic energy.**

(5) When light is incident on metals like calcium, potassium, rubidium etc., then electrons are emitted. Photo-electric cell is based on this principle. When light is incident on this type of cell current is produced. Here, **light energy is converted into electric energy.**

(6) When two palms are rubbed with each other then heat is generated. In this case, **mechanical energy is converted into heat energy.**

(7) When photographic film is exposed to light, then by chemical reaction photos are produced. Here **light energy is transformed into chemical energy.**

(8) In pharmaceutical industry bacteria and germs are destroyed by ultra- and infra-sound waves and naphtha is dissolved in water. Besides, clothes are cleaned by ultra-sound. Here **sound energy is converted into mechanical energy.**

(9) We know electric bells work by electricity. Telephone also works by electricity. In both cases we hear sound. Here **electric energy is transformed into sound energy.**

(10) When coal is burnt heat is produced. This happens due to chemical action. In this case **chemical energy is transformed into heat energy.**

(11) In electric cell electricity is produced due to chemical reaction. Here **chemical energy is transformed into electric energy.**

When energy is transformed from one form to another form, no loss or gain of energy occurs. That means, it is impossible to create or destroy energy. When one form of energy disappears it reappears in another form. This is called **conservation of energy.** There is a principle about it. It is called **conservation principle of energy. It is also called conservation law of energy.**

Formation of a model : Put a rectangular solar panel in front of a cap on the head. Insert a point along with the solar panel through wire and electronic connection for charging a mobile phone. While moving keeping the cap on head the mobile phone will be charged by solar energy [Fig. 5·12]. Solar energy is getting converted to electric energy.

5·6·2 Unit of energy

Since energy is measured with work, so unit of energy and work is the same. That means, unit of energy in S. I. system is Joule (J).

5·6·3 Dimension of energy

Dimension of energy and work is the same, $[E] = [ML^2T^{-2}]$

When a body is moving it gains kinetic energy. For example, a body of mass m moving with velocity v acquires kinetic energy of $\frac{1}{2}mv^2$.

Most common form of energy is the mechanical energy. Energy that remains in a body due to position or motion is called mechanical energy. Mechanical energy is of two types, viz— (i) Kinetic energy and (ii) Potential energy. We will discuss these in this chapter.

Fig. 5·12

Do yourself : Push a book on your reading table towards a pen with strong force. What will you see ? You will see that the pen has become mobile. Why ? Explain.

In this case, capacity to do work i.e., kinetic energy was generated in the pen. So the pen moved forward.

Perceptional work : Whether the power of a body moving with uniform velocity depends on velocity or not ?

According to Newton's first law to keep a body moving with uniform velocity the force needed is zero. So, power of a body moving with uniform velocity is zero which does not depend on velocity.

5·7 Kinetic energy

When a nail is struck by a hammer strongly on a wall the nail enters inside the wall overcoming the resistance of the wall. The hammer can do this work because of its motion i.e., due to the kinetic energy of the hammer the nail can overcome the resistance of the wall.

You have seen sailing boats in the river. Kinetic energy of the current in the river drives away the boat. When wind flows strongly, then the boat can move fast if sail is hoisted. Using the kinetic energy of wind the boat can move fast by hoisting the sail.

While coming down on the plane from hill, current in the river flows strongly. Due to strong kinetic energy of the current large pieces of stones are rolled by the river.

Again, competitors while making high jump and long jump do not jump from rest, but they jump after running a certain distance from behind. As a result, they can go a long distance by jump.

From all the above events it is observed that while stopping a moving body by applying external force, the total amount of work done by the body before coming to rest is the measure of kinetic energy of the body.

Energy possessed by a body by virtue of its motion is called kinetic energy. Any moving body has kinetic energy.

Unit : Unit of kinetic energy and work is the same. That means, unit of kinetic energy is joule.

Dimension :

$$[E_k] = [\frac{1}{2}mv^2]$$
$$= [M]\,[LT^{-1}]^2$$
$$= [ML^2T^{-2}]$$

Motion of a body can be translational and rotational or complex motion consisting of translational-rotational. So, kinetic energy of a body can be either translational kinetic energy or rotational kinetic energy or both. Motion of a freely falling body is a translational kinetic energy. Kinetic energy of a rotating fan is rotational kinetic energy. In the motion of a wheel or a football there are both translational and rotational kinetic energies.

Examples :

(1) When a stone is kept touching a glass nothing happens, but when the stone is thrown on the glass, it breaks. The stone gets this ability due to motion.

(2) When a hammer strikes a nail strongly on a wall, the nail penetrates the wall. Due to the motion of the hammer it can do this work. That means, due to kinetic energy of the hammer it can overcome the resistance of the wall.

(3) Currents in the river become very strong while coming down from a hill on the plane. As the kinetic energy of the current is very strong so large pieces of stones are driven or rolled through the currents.

Do yourself : Float a boat with sail and another boat without sail. If there is a strong wind, what are you going to see ? You will see that the boat having sail moves faster than the boat without sail. Explain it.

Kinetic energy of current of the river drives away the boat. Using kinetic energy of wind a boat can move faster by sailing.

5·7·1 Derivation of equation for kinetic energy

In case of translational motion :

Amount of work done by a moving body before coming to rest is the measure of kinetic energy.

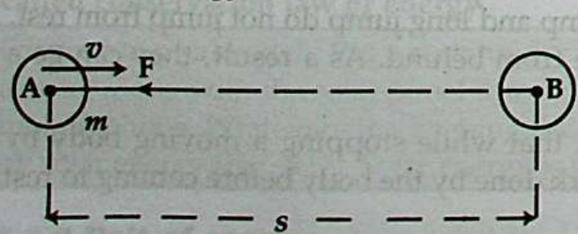


Fig. 5·13

Suppose, a body of mass 'm' is moving along AB with a velocity v. A constant force F is applied opposite to the motion along BA. Due to this uniform retardation will be produced. Let the uniform retardation = a and the body came to rest at point B after covering a distance s from point A. Here, final velocity, $v = 0$.

$\therefore$ **Kinetic energy** = work done before coming to rest
= force × distance travelled before coming to rest
= $F \times s$

We know from Newton's second law of motion that,
Force = mass × acceleration or retardation

$\therefore \; F = ma$

According to description, $0 = v^2 - 2as$

or, $2as = v^2$ or, $s = \dfrac{v^2}{2a}$

Putting the values of F and s in the above equation, we get,

Kinetic energy $= ma \times \dfrac{v^2}{2a} = \frac{1}{2}mv^2$ **or, K. E.** $= \frac{1}{2}mv^2$

That means, **kinetic energy (K. E.)** $= \frac{1}{2}mv^2 = \frac{1}{2} \times$ **mass × (velocity)²** ... (5.26)

Calculus method : Suppose due to the action of applied variable force F on a body a displacement ds occurs along the direction of the force. So, work done by the applied force is,

$$dW = Fds$$

$$= mads \qquad \left[\because F = ma \text{ and } a = \frac{dv}{dt}\right]$$

$$= m\frac{dv}{dt}ds = m\frac{dv}{ds} \times \frac{ds}{dt} \times ds$$

$$= m\frac{ds}{dt} \times \frac{dv}{ds} \times ds = mvdv \qquad \left[\because v = \frac{ds}{dt}\right]$$

$$\therefore dW = mvdv \qquad \ldots \qquad \ldots \qquad \ldots \qquad (5.27)$$

Kinetic energy of the body is measured by the total work done by the applied force when the velocity of the body increases from zero to v. So, by integrating the equation (5·27) from unit 0 to v, we get,

$$\text{Kinetic energy, } E_k = W = \int_0^v dW = m\int_0^v vdv = m\left[\frac{v^2}{2}\right]_0^v$$

$$= \frac{1}{2}m(v^2 - 0)$$

$$\therefore E_k = \frac{1}{2}mv^2, \text{ here } m = \text{constant} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (5.28)$$

$\therefore$ Kinetic energy $= \frac{1}{2} \times$ mass × (velocity)²

It is the equation of kinetic energy.

From the above equation we can come to the following conclusions :

- Kinetic energy at any moment is equal to half the product of mass of and the square of the velocity of the body at that moment.
- Kinetic energy of a body of a fixed mass, $E_k \propto v^2$ i.e., proportional to the square of velocity.
- Kinetic energy $= \frac{1}{2}\dfrac{(\text{momentum})^2}{\text{mass}}$

Relation between kinetic energy and momentum :

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}\times\frac{m^2v^2}{m} = \frac{1}{2}\frac{(mv)^2}{m}$$

$$= \frac{1}{2}\frac{P^2}{m} \qquad [\because \text{momentum, } P = mv]$$

$$\therefore \text{Kinetic energy} = \frac{1}{2}\times\frac{(\text{momentum})^2}{\text{mass}}$$

5·8 Work-energy theorem

Work done by the acting resultant force on a body is equal to the change of its kinetic energy.

Derivation : Suppose a body of mass m is moving with initial velocity v_0. If a force of fixed magnitude is applied on the body along its direction of motion, then velocity of the body will increase. As a result the body will acquire energy. Let the final velocity be v after travelling a distance s. Then work done is, $W = F \times s$.

acceleration produced by the force,

$$a = \frac{F}{m} = \frac{v^2 - v_0^2}{2s} \qquad [\because v^2 = v_0^2 + 2as]$$

$$\text{or, } F = ma = m\left(\frac{v^2 - v_0^2}{2s}\right)$$

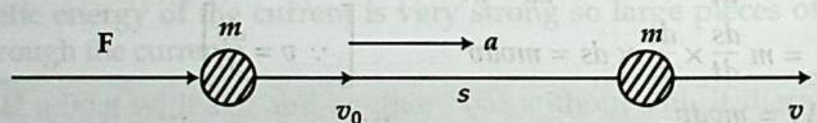


Fig. 5·14

$$\therefore \text{work done, } W = F \times s = m\left(\frac{v^2 - v_0^2}{2s}\right)\times s = \frac{1}{2}m(v^2 - v_0^2)$$

$$\therefore \quad W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \quad \ldots \quad \ldots \quad (5.29)$$

= Final kinetic energy − Initial kinetic energy

∴ **Work doen by the force = acquired energy = change of kinetic energy**

Hence, increase of kinetic energy of an object is equal to the work done by the applied force. It is the work-energy theorem. Equation (5.29) proves the theorem.

[N. B. The theorem is applicable for variable force.]

Alternative method :

Let a particle of mass m travel from point A to point B through path AB. A very small section of this path AB has been denoted by $d\vec{s}$ [Fig. 5·15]. If the force $\vec{F}$ is acting during the displacement of the particle by $d\vec{s}$, then work done,

$$dW = \vec{F}\cdot d\vec{s}$$

So, work done for the entire path AB,

$$W = \int_A^B dW = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B m\vec{a} \cdot d\vec{s} \quad [\because \vec{F} = m\vec{a}]$$

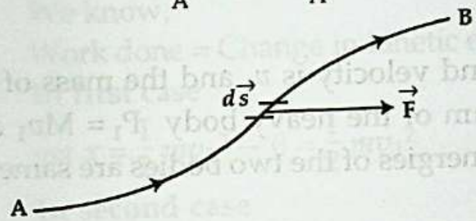


Fig. 5·15

$$\text{or,}\ W = m\int_A^B \frac{d\vec{v}}{dt} \cdot d\vec{s} = m\int_A^B d\vec{v} \cdot \frac{d\vec{s}}{dt}$$

$$= m\int_A^B d\vec{v} \cdot \vec{v} = m\int_A^B v\,dv$$

If the velocity of the particle at points A and B are respectively v_a and v_b, then

$$W = m\int_{v_a}^{v_b} v\,dv = \frac{1}{2}m\,[v^2]_{v_a}^{v_b}$$

$$= \frac{1}{2}m\,(v_b^2 - v_a^2) = \frac{1}{2}mv_b^2 - \frac{1}{2}mv_a^2$$

That means, work done = change of kinetic energy of the particle.

This relation is the work-energy or work-kinetic energy theorem.

It is to be mentioned, whether the force is constant or variable work done is always equal to the change of kinetic energy of the particle.

Verify the work : Competitors do not make high jump or long jump from rest, they jump after running a certain distance from behind. As a result, long distance can be covered by jump. Explain.

Solution of problems

1. Can the kinetic energy be negative ?

If the mass of a moving body is m and velocity v, then kinetic energy of the body $\frac{1}{2}mv^2$. Mass of a body can never be negative. Velocity of a body can either be positive or negative, but square of velocity is always positive. Hence kinetic energy of a body can never be negative.

2. A light body and a heavy body have equal momentum. Which one has higher kinetic energy ?

Let the mass of the heavy body be M and velocity be v_1, whereas the mass and velocity of the light body respectively are m and v_2. If the momentum of the two bodies is same, then

$$Mv_1 = mv_2 = P$$

$$\therefore \frac{\text{Kinetic energy of the light body}}{\text{Kinetic energy of the heavy body}} = \frac{\frac{1}{2}mv_2^2}{\frac{1}{2}Mv_1^2} = \frac{P^2/2m}{P^2/2M} = \frac{M}{m}$$

$\therefore$ If M is greater than m $(M > m)$, then kinetic energy of the light body will be higher than that of the heavy body.

3. A light body and a heavy body have equal kinetic energies. Which one has higher momentum ?

Suppose the mass of the heavy body is M and velocity is v_1 and the mass of the light body is m and velocity v_2. So, the momentum of the heavy body $P_1 = Mv_1$ and momentum of the light body $P_2 = mv_2$. But kinetic energies of the two bodies are same.

$$\therefore \quad \frac{1}{2} Mv_1^2 = \frac{1}{2} mv_2^2$$

$$\therefore \quad \frac{P_1^2}{2M} = \frac{P_2^2}{2m}$$

$$\therefore \quad \frac{P_1}{P_2} = \sqrt{\frac{M}{m}}$$

If M is heavier than m $(M > m)$, then momentum of the heavier body will be larger than that of the lighter body.

Mathematical examples

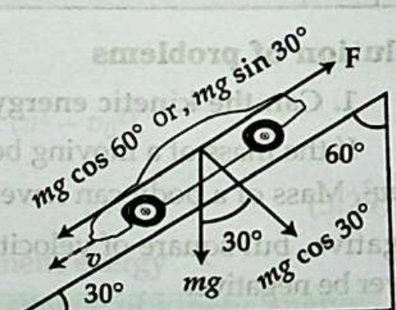
1. A car of mass 2000 kg while descending with a velocity 16 ms^{-1} along a road inclined to 30° with the horizontal, the driver brought it to rest within 40 m by applying brake. How much resisting force was applied on the car ?

According to the problem, the component of gravitational force mg along the plane = $mg \sin 30°$

The resisting force F acts against it.

The resultant of the two forces = F $- mg \sin 30°$

We know, kinetic energy = work



Now, according to work-energy theorem

$$\frac{1}{2} mv_0^2 = (F - mg \sin 30°) \times s$$

$$\therefore \quad \frac{1}{2} \times 2000 \times (16)^2 = \left(F - 2000 \times 9{\cdot}8 \times \frac{1}{2}\right) \times 40$$

$$\text{or,}\ F = \frac{2000 \times (16)^2}{2 \times 40} + 2000 \times 9{\cdot}8 \times \frac{1}{2}$$

$$= 16200 \text{ N}$$

Here,

Mass = 2000 kg

Initial velocity, $v_0 = 16$ ms^{-1}

Distance, $s = 40$ m

$g = 9{\cdot}8$ ms^{-2}

2. A bullet from a rifle can penetrate a piece of wood. If the velocity of the bullet is increased three times, how many pieces of wood of that thickness can be penetrated?

Let the mass of the bullet = m

We know,

Work done = Change in kinetic energy

In first case

$ma\ x = \frac{1}{2} mv_1^2 - 0 = \frac{1}{2} mv_1^2$

In second case

$ma.nx = \frac{1}{2} mv_2^2 - 0$

$= \frac{1}{2} m\ (3v_1)^2 = \frac{9}{2} mv_1^2$

Here,

Thickness of 1 piece of wood = x

Number of pieces of wood = n

$\therefore$ Thickness of n pieces of wood = nx

Initial velocity of the bullet = v_1

Final velocity = $v_2 = 3v_1$

So, $\frac{ma\ x}{ma\ .\ nx} = \frac{\frac{1}{2} mv_1^2}{\frac{9}{2} mv_1^2}$

or, $\frac{1}{n} = \frac{1}{9}$

$\therefore \quad n = 9$

Alternative method :

In the first case, $\frac{1}{2}mv^2$ = work = mgx ... ... (i)

In the second case, $\frac{1}{2}m(3v)^2 = mg \times nx$ ... ... (ii)

Dividing equation (ii) by equation (i) we get,

$$\frac{\frac{1}{2}m\ 9v^2}{\frac{1}{2}mv^2} = \frac{mgnx}{mgx}$$

$\therefore\ n = 9$

So, the number of pieces of wood is 9.

3. If the momentum of a truck weighing 2000 kg is 200 kg ms^{-1}, what is its kinetic energy ?

We know,

$$E_k = \frac{P^2}{2\ m} = \frac{(200)^2}{2 \times 2000}$$

$= 10$ J

Here,

m = 2000 kg

P = 200 kg ms^{-1}

E_k = ?

4. A body of mass of 2 kg is dropped from the roof of a building of height 30 m. Calculate (i) initial potential energy of the body (ii) the velocity at which the body touches the ground (iii) maximum kinetic energy of the body (iv) kinetic and potential energy of the body at a height of 3m.

(i) initial potential energy of the body = $mgh = 2 \times 9{\cdot}8 \times 30 = 588$ J

(ii) let the body touch the ground with velocity v.

Now potential energy of the body when it is on the roof = kinetic energy of the body when it touches the ground i.e., $mgh = \frac{1}{2} mv^2$

$\therefore \quad 588 = \frac{1}{2} \times 2 \times v^2$

or, $v^2 = 588$

$\therefore \quad v = \sqrt{588} = 24{\cdot}25 \text{ ms}^{-1}$

(iii) maximum kinetic energy of the body = initial potential energy of the body

So, maximum kinetic energy of the body = 588 J

(iv) potential energy of the body at a height of 3m = $2 \times 9{\cdot}8 \times 3 = 58{\cdot}8$ J

kinetic energy of the body at that position = decrease weighing of potential energy = $588 - 58{\cdot}8 = 529{\cdot}2$ J

5. A boy and a man are running simultaneously. The mass of the boy is half of the mass of the man and kinetic energy of the man is half of that of the boy. If the man increases his velocity by 1 ms⁻¹, then his kinetic energy becomes equal to that of the boy. Calculate their initial velocity. [R. B. 2011, 2003; S. B. 2003]

Here,
mass of the boy = m_1
mass of the man, $m_2 = 2m_1$
initial velocity of the boy, $v_1 = ?$
initial velocity of the man, $v_2 = ?$
final velocity of the man, $v_2' = v_2 + 1$

We get from the equation of kinetic energy,
kinetic energy of the boy,

$$KE_1 = \frac{1}{2} m_1 v_1^2 \quad \dots \quad \dots \quad \text{(i)}$$

and kinetic energy of the man,

$$KE_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \cdot 2m_1 v_2^2 = m_1 v_2^2 \quad \dots \quad \dots \quad \text{(ii)}$$

According to the question, kinetic energy of the man = $\frac{1}{2}$ (kinetic energy of the boy)

$$m_1 v_2^2 = \frac{1}{2} \left(\frac{1}{2} m_1 v_1^2\right)$$

$$\therefore \quad 2m_1 v_2^2 = \frac{1}{2} m_1 v_1^2 \quad \dots \quad \dots \quad \text{(iii)}$$

Again, if $v_2' = v_2 + 1$, then according to the question, $\frac{1}{2} m_1 v_1^2 = m_1 (v_2 + 1)^2$

by inserting the value of $\frac{1}{2} m_1 v_1^2$ obtained from equation (iii), we get

$$2m_1 v_2^2 = m_1 (v_2 + 1)^2$$

or, $2v_2^2 = v_2^2 + 2v_2 + 1$

or, $v_2^2 - 2v_2 - 1 = 0$

$$\therefore \quad v_2 = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$$

As velocity is positive, so, $v_2 = 1 + \sqrt{2} = 2{\cdot}41 \text{ ms}^{-1}$

From equation (iii) we get,

$$\frac{1}{2} m_1 v_1^2 = 2\, m_1 v_2^2$$

or, $v_1^2 = 4 \times (2{\cdot}41)^2$

or, $v_1 = \sqrt{23{\cdot}2324}$

$\therefore \quad v_1 = 4{\cdot}82 \text{ ms}^{-1}$

Ans. Initial velocity of the boy $4{\cdot}82 \text{ ms}^{-1}$ and initial velocity of the man $2{\cdot}41 \text{ ms}^{-1}$

6. A bomb weighing 500 g was dropped from a height of 1 km. What will be its kinetic energy just before touching the ground ?

We know,

$$E_k = \frac{1}{2} mv^2 \quad \dots \quad \dots \quad \text{(i)}$$

or, $v^2 = v_0^2 + 2gs$

$= 0 + 2 \times 9{\cdot}8 \times 10^3$ m

$= 19600 \text{ m}^2\text{s}^{-2}$

$\therefore \quad E_k = \frac{1}{2} \times 0{\cdot}5 \times 19600 = 4900$ J

Here,

mass, $m = 500 \text{ g} = 0{\cdot}5$ kg

displacement, $h = 1 \text{ km} = 10^3$ m

$E_k = ?$

$v_0 = 0$

7. A bomb of mass 1 kg was dropped from a bomber at a height of 1 km. What was its kinetic energy just before touching the ground ?

We know, if the velocity of the bomb just before touching the ground is v, then

$$K = \frac{1}{2} mv^2$$

but, $v^2 = v_0^2 + 2gh$

or, $v^2 = 0 + 2 \times 9{\cdot}8 \times 10^3$

$= 19600 \text{ m}^2\text{s}^{-2}$

$\therefore \; K = \frac{1}{2} mv^2 = \frac{1}{2} \times 1 \times 19600$

$= 9800$ J

Here,

mass, $m = 1$ kg

height, $h = 1 \text{ km} = 10^3$ m

kinetic energy, K = ?

5·9 Potential energy

Energy that a body acquires due to its position or, energy acquired by the body due to the change of position of the particles within the body is called the potential energy of the body.

Suppose a piece of brick is kept above the roof, or water is pumped in a tank placed on the roof. In both the cases, the brick and water have acquired some energy. This type of energy is called **potential energy.** Potential energy of a body depends on the mass of the body, height of it from the ground and acceleration due to gravity at the place of observation.

Examples :

(a) A spring is attached in a toy motor-car [Fig. 5·16]. When you wind this spring, its size becomes smaller. We do work to change this size which remains stored as potential energy. But as the wind is released, the spring regains its original shape. Wheels of the toy are attached to the spring. So, the wheels start rotating i.e., due to potential energy the spring works to make the toy move.

Fig. 5·16

(b) In a wrist watch a wheel is attached along an elastic spring [Fig. 5·16]. While winding this spring, its size gets shrinked or contracted. We do work to change the size of the spring which remains stored as potential energy in the spring. There is connection of the hand of the watch with the spring, so the spring rewinds and rotates in opposite direction. As a result the hand of the watch rotates. The potential energy of the spring transforms into kinetic energy.

Similarly, when a metal plate is pulled, or a rubber is expanded— in all cases due to change of shape potential energy is stored.

(c) Potential energy remains stored due to change of position in water above the ground, ice above the hill and cloud in the sky.

Potential energy is measured by the work that is done when the body returns to its initial normal position or configuration from the present position or configuration.

Types of potential energy

There are different types of potential energy, viz—

(1) Gravitational potential energy

(2) Elastic potential energy

(3) Electric potential energy

5·9·1 Mathematical expression for potential energy

1. Gravitational potential energy

To raise a body above the ground against the gravitational force an external source or agent is needed to do work. This work remains stored in the body as potential energy. This energy is called gravitational potential energy. In this case, surface of the earth is considered as the **reference level.** Let us now measure the potential energy.

Calculus method : Let an object of mass m be raised above the surface of the earth to a height dh against the force due to gravity.

Work done due to this is,

$$dW = \vec{F} \cdot d\vec{h}$$

$$\text{or,} \quad dW = Fdh \qquad \ldots \quad \ldots \quad \ldots \quad (5.30)$$

Here F = force applied by external source, and dh = vertical height. The angle between F and dh is zero.

In order to raise a body above, a vertical force equal to the weight of the body is to be applied.

∴ **Applied force, F = Weight of the body**

$= mg$

So, total work done in raising the body to a height h at position A [Fig. 5·17] is the summation of the small work done as in equation (5.30).

∴ Gravitational potential energy = total work done in raising the body to the height h above the ground

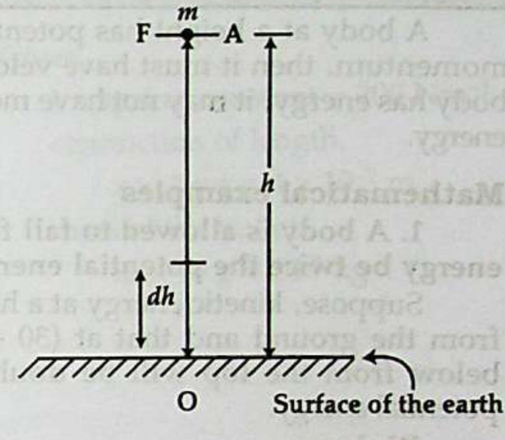


Fig. 5·17

$$P.\,E. = \int_0^h F dh = \int_0^h mg dh$$

For small height g is constant (assumed) and we can write,

$$P.\,E. = mg\int_0^h dh = mg\,[\,h\,]_0^h = mg\,[\,h - 0\,] = mgh$$

That means, gravitational potential energy.

$$P.\,E. = mgh \qquad \ldots \qquad \ldots \qquad (5.31)$$

i.e., **Gravitational potential energy = mass × acceleration due to gravity × height above the reference level.**

[N. B. As the body descends h bcomes less and less, so also the gravitational potential energy. As on the surface of the earth, $h = 0$, so the gravitational potential energy also becomes zero.]

Value of gravitational potential energy of a body depends on the position of the body with respect to the reference plane. Potential energy of a place considering the reference level as the sea level and potential energy of that place considering the reference level as the hill-top, not be same but different. Actually absolute value of potential energy of a place can never be determined, only change of potential energy of a place with reference to a plane or standard plane can be determined.

Potential energy can either be positive or negative. It depends on the reference level. If the reference level is the ground, then at a point above the ground it will be positive. On the other hand inside a mine or inside the earth potential energy is negative.

Work : Take two tanks full of water which have similar size of outlet tube. Keep one tank on the ground and the other one on the roof of the building. Now open the two outlets. Water of which tank would have higher velocity ?

Since the tank on the roof is in a higher place, it gains potential energy. Hence if the outlet tubes are opened, water from the tank on the roof will come out with more velocity.

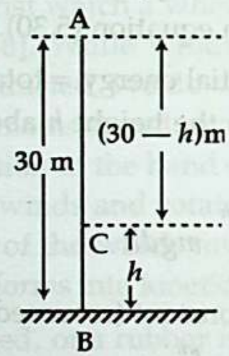
Inquisitive work : A body has energy but has no momentum or has momentum but no energy—is it possible to occur like this ?

A body at a height has potential energy but no momentum. Again, if a body has momentum, then it must have velocity. So, that body will have kinetic energy. So, if a body has energy, it may not have momentum, but if it has momentum, then it must have energy.

Mathematical examples

1. A body is allowed to fall freely from a height of 30 m. Where will its kinetic energy be twice the potential energy ? [R. B. 2010; J. B. 2006 ; B. B. 2003]

Suppose, kinetic energy at a height h from the ground and that at $(30 - h)$ m below from the top will be double the potential energy.



Here,
Height, $h = 30$ m
Initial velocity, $v_0 = 0$
Acceleration, g

We know,

Potential energy, $E_p = mgh$

Kinetic energy, $E_k = \frac{1}{2}mv^2$

According to the question,

$E_k = 2E_p$ ... ... (i)

Here, $v^2 = v_0^2 + 2g(30 - h)$

or, $v^2 = 0 + 2g(30 - h) = 2g(30 - h)$

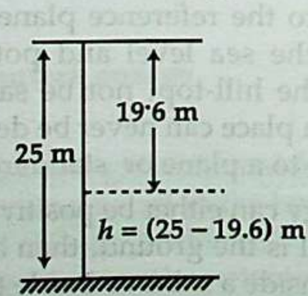
$\therefore\ E_k = \frac{1}{2}\ m \times 2g(30 - h) = mg(30 - h)$

According to equation (i)

$mg(30 - h) = 2mgh$

$\therefore\ 2h = 30 - h$ or, $h = 10$ m

2. A 4 kg mass is allowed to fall freely from a height of 25 m at the gravitational attraction. After 2 seconds what will be the kinetic energy and potential energy of the mass ?



Here,
$m = 4$ kg
$g = 9{\cdot}8\ \text{ms}^{-2}$
$t = 2$ s

We know,

after 2 sec, $h = v_0t + \frac{1}{2}gt^2$

$= 0 + \frac{1}{2} \times 9{\cdot}8 \times 4$

$= 19{\cdot}6$ m

$v^2 = v_0^2 + 2gh = 0 + 2 \times 9{\cdot}8 \times 19{\cdot}6$

$= 2 \times 9{\cdot}8 \times 19{\cdot}6$

$\therefore$ after 2 sec kinetic energy

$E_k = \frac{1}{2}\ mv^2 = \frac{1}{2} \times 4 \times 2 \times 9{\cdot}8 \times 19{\cdot}6 = 768{\cdot}32$ J

Potential energy, $E_p = mg(25 - 19{\cdot}6) = 4 \times 9{\cdot}8 \times 5{\cdot}4 = 211{\cdot}68$ J

3. By contracting the spring of a gun by a 4 cm bullet of mass 10 g is fired. When the spring returns to its equilibrium position, then what is the velocity of the bullet just released ? (spring constant is 200 Nm^{-1})

Here, kinetic energy of the contracted spring $= \frac{1}{2} kx^2$

kinetic energy of the bullet $= \frac{1}{2} mv^2$

According to the question, $\frac{1}{2} kx^2 = \frac{1}{2} mv^2$

or, $kx^2 = mv^2$

or, $v^2 = \frac{kx^2}{m}$

$= \frac{200 \times (4 \times 10^{-2})^2}{10^{-2}} = 32$

$\therefore \quad v = 5{\cdot}657 \text{ ms}^{-1}$

Given,

k = spring constant = 200 Nm^{-1}

contraction of length,

$x = 4 \text{ cm} = 4 \times 10^{-2} \text{ m}$

mass of the bullet,

$m = 10 \text{ g} = 10^{-2} \text{ kg}$

4. Show that in case of a falling body gain of kinetic energy due to travelling a fixed distance, the potential energy is lost by the same amount.

Suppose, kinetic energy = T

potential energy = V

total energy, E = T + V ... ... (i)

Further suppose, due to travelling a fixed distance gain of kinetic energy is ΔT change of potential energy is ΔV

$\therefore \quad T + \Delta T + V - \Delta V = E$ ... ... (ii)

by subtracting equation (i) from equation (ii) we get,

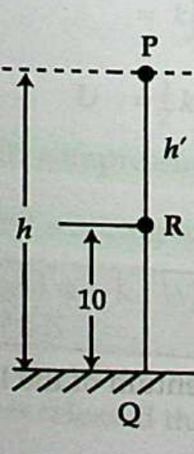
$\Delta T - \Delta V = 0$

or, $\Delta T = \Delta V$

$\therefore$ increase of kinetic energy = decrease of potential energy

5. A body was dropped from a fixed height. If the kinetic energy was twice the potential energy at a height of 10 m, then from what height the body was dropped ?

[J. B. 2006]



Suppose, the body of mass m was dropped from height P and at point R, kinetic energy = 2 × potential energy

Potential energy at point R, $E_p = mgx$

$= mg \times 10 = 10\,mg$ ... (i)

Let the velocity of the body at point R be v

We know,

$v^2 = v_0^2 + 2gh'$

or, $v^2 = 2g(h - x)$ $\quad [\because v_0 = 0]$

$= 2g(h - 10)$

Kinetic energy at point R, $E_k = \frac{1}{2} mv^2 = \frac{1}{2} m \times 2g (h - 10)$

$= mg (h - 10)$

According to the question,

$mg (h - 10) = 2 \times 10\, mg = 20\, mg \quad \dots \quad \dots \quad (ii)$

$\therefore \; h - 10 = 20$

or, $h = 20 + 10 = 30$ m

Ans. height is 30 m

6. A stationary car of mass 50 kg acquires velocity of 15 ms^{-1} after 2 s due to the action of a fixed force. Calculate the applied force on it and what will be its kinetic energy after 4 s ?

Here,

$m = 50$ kg

$v = 15$ ms^{-1}

$v_0 = 0$

$t = 2$ s

acceleration, $a = ?$

force, F = ?

$t_1 = 4$s

after 4 s velocity, $v_1 = ?$

after 4 sec kinetic energy, K = ?

We know,

$F = ma$

Again, $v = v_0 + at$

or, $15 = 0 + a \times 2$

$\therefore \; a = \frac{15}{2} = 7{\cdot}5$ ms^{-2}

$\therefore$ F = $ma = 50 \times 7{\cdot}5 = 375$ N

Again, $v_1 = v_0 + at$

$= 0 + 7{\cdot}5 \times 4 = 30$ ms^{-1}

Kinetic energy, $K = \frac{1}{2} mv_1^2 = \frac{1}{2} \times 50 \times (30)^2 = 22500$ J

Perceptional work : Kinetic energy of a light body and a heavy body is same. Which of the two bodies has higher momentum ?

Suppose, mass and velocity of the lighter body = m and v respectively and mass and velocity of the heavier body = M and V respectivley.

According to question,

$$\frac{1}{2} mv^2 = \frac{1}{2} MV^2$$

or, $mv^2 = MV^2$

or, $\frac{v^2}{V^2} = \frac{M}{m}$

or, $\frac{V}{v} = \sqrt{\frac{m}{M}}$

$$\therefore \; \frac{\text{momentum of the heavier body}}{\text{momentum the lighter body}} = \frac{MV}{mv}$$

$$= \frac{M}{m}\sqrt{\frac{m}{M}} = \sqrt{\frac{M}{m}} > 1$$

Hence, momentum of the heavier body is larger than the momentum of the lighter body.

2. Elastic potential energy

Within elastic limit, if a force is applied on a body it becomes strained work is done to produce strain in a body. This work remains stored as potential energy. This energy is called **elastic potential energy.** It will be easier to understand the potential energy created in a spring from the following discussion.

Potential energy of a stretched spring : Let one end of an ideal horizontal spring be fixed at a wall and a body of mass m be attached to the other end. The body can move on a frictionless horizontal surface [Fig. 5.18]. We ignore the mass of the spring compared to the mass of the body.

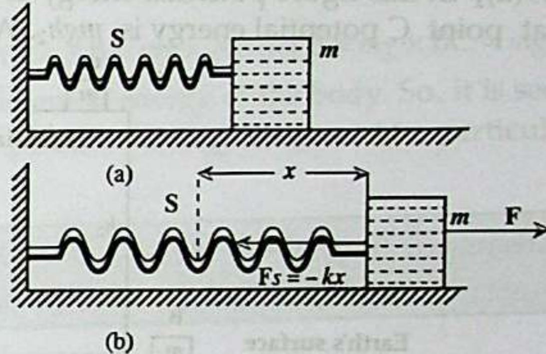


Fig 5.18

Now, the spring is stretched along its length by a distance x. Due to elastic property the spring will exert a force called restoring force which will try to oppose the deformation. This restoring force is proportional to the displacement and acts opposite to the displacement i.e.,

$$F \propto -k$$

or, $F = -kx$

where k is a constant of proportionality and is called force constant of the spring.

If the spring is strained along its length by pulling the body, then a restoring force will be generated in the spring against the applied force due to elastic property. If the length of the spring from left to right is increased by an amount x along the horizontal due to the application of force, then a restoring force of $-kx$ will be generated in the spring. Now, in order to displace the body by a distance x work is to be done by applying an equal and opposite force $F = kx$. Work done in this expansion by the applied force will be the stored energy in the spring.

So, potential energy

$$U = W = \int_0^x F dx = \int_0^x kx \, dx$$

$$= k\int_0^x x \, dx = k\left[\frac{x^2}{2}\right]_0^x$$

$$\therefore \quad U = \frac{1}{2}kx^2 \quad \ldots \quad \ldots \quad \ldots \quad (5.32)$$

While compressing the spring by x, stored potential energy will be $\frac{1}{2}kx^2$.

Here k = spring constant $= \frac{F}{x}$.

Perceptional work : What do you understand by the restoring force at any moment of a spring of 15 N ?

Restoring force of 15 N of a spring means that if the spring is pulled by a force of 15 N and is released then it returns to the previous state by the same force.

Solution of problems

1. Can potential energy be negative ?

Direction : Potential energy can be negative. For example, potential energy at any point above earth's surface is positive. Potential energy inside a mine is negative [Fig. 5·19(a)]. In this figure potential energy at point B on the earth's surface is zero. At height h_2 at point C potential energy is mgh_2. While coming down from point C to B potential

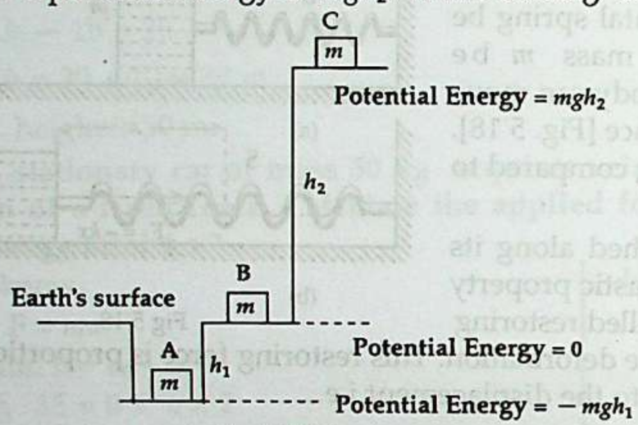


Fig. 5·19(a)

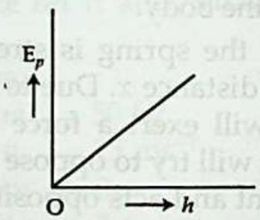


Fig. 5·19(b)

energy starts decreasing. Similarly going from point B to bottom point A potential energy will decrease. Since potential energy at point B is zero, so potential energy at point A will be negative. If A is at depth h, then potential energy at that point will be $-mgh_1$. If the body is to be taken again from point A to B, work is to be done against the weight of the body or against force of gravity. Relation between height and potential energy has been shown in fig. 5·19(b).

2. How does the potential energy of a body become zero ?

Direction : Work is to be done against a body to take it from the standard position or shape to any other position or shape. This work remains stored in the body as potential energy. While coming back to its original position the body can do work, so potential energy of the body decreases and becomes zero when it reaches to its standard position. At this stage the body does not do any work.

3. Why does gravitational potential energy depend only on h but not on the path ?

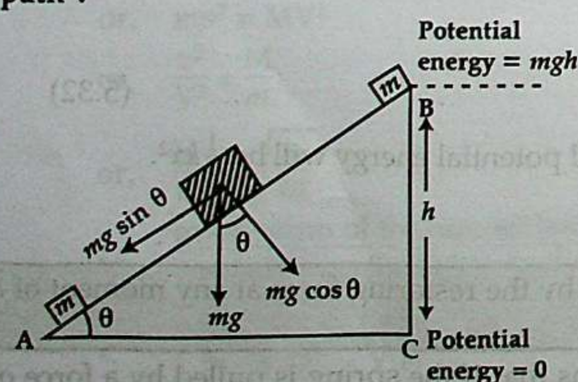


Fig. 5·20

Direction : If a body is taken to height h vertically, its potential energy does not depend on the path. That means not taken to height h vertically if the body is taken to that height through a different path it will have same potential energy. For example, a body of mass m is taken from point A to B through an inclined plane AB, potential energy of the body will be mgh. If that body is taken vertically from point C to B, potential energy will be mgh [Fig. 5·20].

Suppose the same body of mass m is taken from A to B through a smooth inclined plane AB to a height h. Component of the weight mg of the body along the inclined plane is $mg \sin \theta$. In order to pull the body along this inclined plane work against this force is to be done. Other component of the weight of the body is $mg \cos \theta$ which acts perpendicular to displacement of the body; hence no work is done. Displacement of the body along the inclined plane is AB. So,

Total work = force × displacement = $mg \sin \theta \times AB = mg \times AB \sin \theta = mg \times BC = mgh$

According to definition, this work is the potential energy of the body. So, it is seen that whatever way the body is taken to point B potential energy is same. At a particular height gravitational potential energy is same.

Calculate : In order to raise a body of weight 50 N to a height of 6 m a lift was used. It uses 70 J energy. Calculate the dissipated energy.

Here, used energy = work = force × displacement
= weight × height = 50 × 6 = 300 J

Dissipated energy = supplied energy − used energy
= 300 J − 70 J = 230 J

Work : Keep some glass marbles of equal mass on a horizontal table in the same row touching each other. Rolling similar two marbles strike one end of that row. What will you see ? Why two marbles from the other end will be in motion at the same time with same velocity ?

In this case both the momentum and mechanical energy follow conservation principle. As a result two marbles from the other end of the row become mobile at the same time with same velocity.

Mathematical examples

1. An object of mass of 2 kg is 15 m above the ground. When it is thrown it hits the ground at a velocity of 10 ms^{-1}. What frictional force will act on the object while falling ?

Here, potential energy = Energy spent in friction + final kinetic energy

or, $mgh = Fh + \frac{1}{2}mv^2$

$mgh = mgh + \frac{1}{2}mv^2 \quad [\because F = mg]$

$Fh = mgh - \frac{1}{2}mv^2 = 2 \times 9{\cdot}8 \times 15 - \frac{1}{2} \times 2\,(10)^2$

$= 294 - 100 = 194$

$\therefore F = \frac{194}{h} = \frac{194}{15} = 12{\cdot}9$ N

Here,
mass, $m = 2$ kg
acceleration due to gravity, $g = 9{\cdot}8$ ms^{-2}
height, $h = 15$ m
velocity, $v = 10$ ms^{-1}

2. A person weighing 60 kg ascends a top of 180 m height in 20 minutes. Calculate work and applied power.

According to question, work against the gravitational force,

W = force × displacement along the line of force

= weight × vertical displacement

$= mg \times h$

Here, $m = 60$ kg, $g = 9{\cdot}8$ ms^{-2}, $h = 180$ m

∴ required work, $W = 60 \text{ kg} \times 9{\cdot}8 \text{ ms}^{-2} \times 180 \text{ m}$

$= 10{\cdot}584 \times 10^4 \text{J}$

∴ applied power, $P = \frac{W}{t} = \frac{10{\cdot}584 \times 10^4 \text{ J}}{20 \times 60 \text{ s}}$

$= 88{\cdot}2$ W

Here, t = 20 minutes = 20×60 s

3. A body of mass of 5 kg falls on a pin from a height of 5 m and the pin enters 10 cm inside the ground. Calculate the average resisting force of the ground.

We know,

potential energy of a falling body = work done against the opposing force

$= Fs = F \times 0{\cdot}1$

total fall of the body $= h + s = 5 + 0{\cdot}1$

$= 5{\cdot}1$ m

Here,
mass of the body, $m = 5$ kg
height, $h = 5$ m
displacement, $s = 10$ cm $= 0{\cdot}1$ m
resisting force, F = ?

∴ potential energy of the body $= mg(h + s)$

$= 5 \times 9{\cdot}8 \times 5{\cdot}1$

according to the question, $F \times 0{\cdot}1 = 5 \times 9{\cdot}8 \times 5{\cdot}1$

∴ F = 2499 N

4, Water falling from a fountain from a height of 250 m on the ground and move horizontally with a fixed velocity. Find the velocity of water flow considering that there is no dissipation of energy.

Potential energy lost due to fall of water on the ground from height of 250 m is transformed into kinetic energy.

Now, let the mass of water be m, velocity v and height h, then we can write,

$$mgh = \frac{1}{2}mv^2$$

or, $v = \sqrt{2gh}$

$= \sqrt{2 \times 9{\cdot}8 \times 250} = 70 \text{ ms}^{-1}$

5·10 Experimental

Name of the experiment : Period : 2	Determination of potential energy of a spring

Theory : Let a weight of mass m be hung at one end of a spring or the spring is expanded by an amount x due to application of force F. This displacement of the spring is proportional to the applied force, i.e., $F \propto x$.

or, $F = Kx$ [here, K = spring constant] ... (i)

Now, work done by external force to expand the spring from position x_1 to x_2 is,

$$W = \int_{x_1}^{x_2} F\,dx$$

$$= \int_{x_1}^{x_2} Kx\,dx = K\int_{x_1}^{x_2} x\,dx$$

$$= K\left[\frac{x^2}{2}\right]_{x_1}^{x^2} = \frac{K}{2}\,(x_2^2 - x_1^2)$$

$$\therefore \quad W = \frac{1}{2}K\,(x_2^2 - x_1^2) \quad \ldots \quad \ldots \quad \ldots \quad \text{(ii)}$$

This work is positive work. This work remains in the spring as potential energy. Let

$$x_1 = 0 \text{ and } x_2 = x \text{, so}$$

$$W = \frac{1}{2}K\,(x^2 - 0)$$

$$\text{or,} \quad W = \frac{1}{2}Kx^2 \quad \ldots \quad \ldots \quad \ldots \quad \text{(iii)}$$

If a spring of mass m expands by l and in this position if the spring is pulled by an amount x and released, then it executes simple harmonic motions. Its time period becomes,

$$T = 2\pi\sqrt{\frac{m}{K}}$$

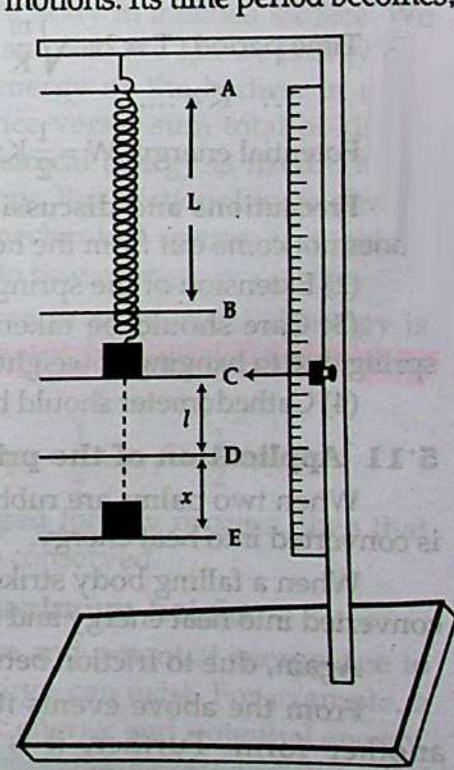


Fig. 5·21

Apparatus :

(1) An experimental spring
(2) One metre scale
(3) Some suitable weights
(4) A hook to hang the spring
(5) A stop watch

Procedure :

(1) The spring is to be hung as per figure.

(2) At one end of the spring if a weight is hung than the spring will be slightly elongated. Now measure the distance between the stationary position and the changed position of the spring by a metre scale. This gives the elongated length l.

(3) Then the weight is pulled downward by a distance x and released. Again, the expansion of length x is measured by the metre scale.

(4) The spring will continue moving ups and down. Time is recorded by a stop watch for 20 complete oscillation. Dividing this time by 20 time period T is to be measured.

(5) By changing weights the processes (3) and (4) are repeated for a few times.

Data Table—1
(Table for the determination of T and x)

No. of observa-tions	Initial length of the spring L(m)	Increase in length after hanging the load l (m)	Extension of spring by applying force for oscillation x (m)	Time for 20 oscillations (sec)	Time period T (sec)	Spring cons-tant K	Potential energy $W = \frac{1}{2}Kx^2$ (J)

Calculations :

$l = \ldots\ldots$ m

$x = \ldots\ldots$ m

Time period, $T = 2\pi\sqrt{\frac{m}{K}}$

$\therefore$ K = ...

Potential energy, $W = \frac{1}{2}Kx^2$ = ... Joule

Precautions and discussion : (1) The spring is to be hanged in such a way so that it does not come out from the hook after hanging the load.

(2) Extension of the spring is to be increased slowly in steps.

(3) Care should be taken so that no hindrance occurs during expansion of the spring due to hanging of weight at the end of the spring.

(4) Cathedometer should be used for measuring accurate length.

5·11 Application of the principle of conservation of energy

When two palms are rubbed together, they become warm; here mechanical energy is converted into heat energy.

When a falling body strikes the ground and get stopped, then mechanical energy is converted into heat energy and some sound energy.

Again, due to friction between different parts of a machine heat energy is created.

From the above events it is observed that energy is converted from one form to another form. Further, it is seen from Einstein's theory of relativity that mass is converted into energy. If energy of a body increases, its mass also increases. On the other hand, if energy decreases in a body, its mass also decreases. When a box is pulled over the floor heat is produced due to friction.

It is seen from all the above cases (conservative or non-conservative) that energy is converted from one form to another form, but is never exhausted or destroyed. **This is the principle of conservation of energy.**

Law : "Energy can neither be created nor destroyed, but can only be converted from one form to another." Total energy of the universe is constant. In electric iron heat is produced when electricity is passed though it. We iron our clothes with this heat. In this case, electric energy is converted to heat energy and finally heat energy is converted to mechanical energy. Here no energy is destroyed, only there is transformation.

You have heard about nuclear reactor. In nuclear reactor nuclear fission is created by bombarding a heavy nucleus $\left(U^{235}_{92}\right)$ by a neutron. In this reaction enormous amount of heat energy is produced. Electricity is produced by rotating turbines by using this heat energy. In this case it is seen that nuclear energy is converted into heat energy and heat energy is converted into electric energy. In this case also there is no loss or destruction of energy. Only energy is transformed from one form to another form.

When energy is changed from one form to another form, then no increase or decrease of energy occurs. That means, either creation or destruction of energy is impossible. When one form of energy is destroyed, it appears in another form. It is called conservation principle of energy.

We can cite innumerable examples of conversion of mechanical energy— for example, oscillation of simple pendulum and motion of a body in inclined surface. We can come to a very important conclusion from these examples. We know, energy can neither be created nor can be destroyed. So, kinetic energy of the bodies, in these examples, are changed to potential energy only and vice versa; sum total of kinetic energy and potential energy is constant i.e., total mechanical energy is fixed. This is called the **principle of conservation of mechanical energy.** But frictional force always retards the motion of a body. So, a portion of total mechanical energy is spent or dissipated to overcome this resistance and is converted into heat energy.

In case of the above examples principle of conservation of mechanical energy is applicable. If there is no dissipated force and if the collision is perfectly elastic, then total energy remains unchanged. Hence,

Kinetic energy before collision = stored potential energy $= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

When the magnitude of a quantity remains unchanged for any process, then that quantity is called conserved. So, total mechanical energy is conserved.

A. Conservation of energy of a body thrown at a maximum height

There appears kinetic energy in a body due to motion and potential energy due to position. In a moving body both kinetic and potential energy can exist. For example, a flying aeroplane or a stone thrown above have both kinetic energy and potential energy. Then total energy of the body means the summation of kinetic energy and potential energy. So, total energy—

$$E_T = E_k + E_p \qquad \ldots \qquad \ldots \qquad \ldots \qquad (5.33)$$

Kinetic energy of a body can be transformed into potential energy or potential energy can be transformed into kinetic energy. Many examples of this type can be cited. Now we will apply principle of conservation of energy of maximum height on a body thrown above.

Let a stone of mass m be thrown vertically above with a velocity v_0 [Fig. 5·22]. If the ground is considered as the reference level, then initial potential energy of the stone = 0 and initial kinetic energy $= \frac{1}{2} m v_0^2$. As the stone moves up its potential energy increases and velocity continues to decrease i.e., kinetic energy continues to decrease. So, during ascending kinetic energy of the stone is converted into potential energy. At height h if the velocity of the stone is v, $(v < v_0)$, then kinetic energy of the stone at that point $= \frac{1}{2} mv^2$ and potential energy $= mgh$. Hence, total energy of the stone $= \frac{1}{2} mv^2 + mgh$. When the stone reaches to the highest position its velocity momentarily becomes zero. Then kinetic energy becomes zero and potential energy is maximum. If the maximum height of the stone is H, then potential energy of the stone at that position = mgH.

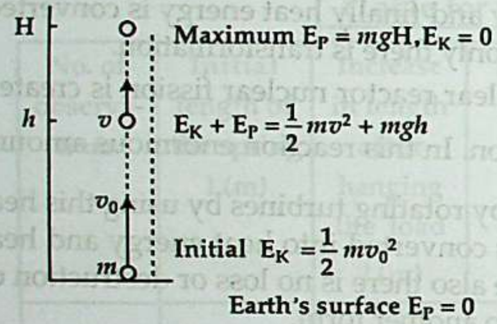


Fig. 5·22

So, at the highest position total kinetic energy of the stone is converted into potential energy.

After reaching the maximum height the stone again starts descending. Then opposite phenomenon occurs; potential energy of the stone then starts decreasing and its kinetic energy starts increasing. Just on the reference level only kinetic energy exists and potential energy becomes zero.

In this case it can be proved easily that if there is no dissipative force like frictional force, then total energy of the stone in the initial state (which is totally kinetic energy) is equal to total energy in the highest position of the stone (which is totally potential energy). That means, $\frac{1}{2} mv_0^2 = mg$.

In previous positions also total energy remains unchanged. So,

$$\frac{1}{2} mv_0^2 = mgH = \frac{1}{2} mv^2 + mgh$$

This principle is also applicable for a freely falling body. From the initial position from where the body was thrown above with velocity v_0, when the body again comes back to that initial position, the velocity becomes v_0. This time the total energy is kinetic. So, its total energy becomes $\frac{1}{2} mv_0^2$. Hence, **thrown body obeys the principle of conservation of energy at the highest position.**

B. Energy of simple harmonic motion

Motion of a simple pendulum is simple harmonic motion. When pendulum oscillates then in some positions its kinetic energy is converted into potential energy and in other positions potential energy is converted into kinetic energy. But the sum total of potential energy and kinetic energy remains constant.

Suppose, the mass of the bob of a simple pendulum is m and its equilibrium position is at 0. Let the bob reach the maximum point B in one direction by travelling a distance A [Fig. 5·23]. At point B, since velocity $v = 0$, its total energy is potential energy. If the force acting on the simple pendulum is F, then $F = -kx$. So at point B, maximum potential energy,

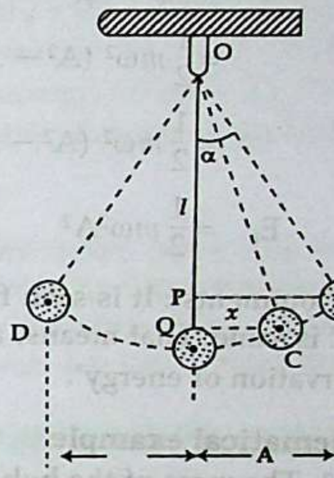


Fig. 5·23

$$E_p = \int_0^A -F\,dx = \int_0^A k\,x\,dx$$

$$= k\left[\frac{x^2}{2}\right]_0^A = \frac{1}{2}kA^2$$

We know,

$$\frac{k}{m} = \omega^2 \quad \therefore \quad k = \omega^2 m$$

$$\therefore \quad E_p = \frac{1}{2} \times m\omega^2 A^2 \quad \ldots \quad \ldots \quad \ldots \quad (5.34)$$

Since at point B kinetic energy of point B, $E_k = 0$, so at point B total energy of the bob

$$E = \frac{1}{2} m\omega^2 A^2 \quad \ldots \quad \ldots \quad \ldots \quad (5.35)$$

Now, let us consider that the bob starting from the position B moves towards the equilibrium position 0 and at one time reaches point C. The distance from the equilibrium position to point C is x and the velocity of the bob is v, then kinetic energy at point C $E_{kC} = \frac{1}{2} mv^2$.

But, in case of simple harmonic motion velocity,

$v = \omega\sqrt{A^2 - x^2}$; so,

$$E_{kC} = \frac{1}{2} m\omega^2 (A^2 - x^2) \quad \ldots \quad \ldots \quad \ldots \quad (5.36)$$

At point C the bob will have some potential energy, whose amount is,

$$E_{pc} = \int_0^x k\,x\,dx$$

$$= k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2}kx^2$$

$$= \frac{1}{2} m\omega^2 x^2 \quad [\because k = m\omega^2] \quad \ldots \quad \ldots \quad \ldots \quad (5.37)$$

Total energy at point C,

$$E_k = E_{kC} + E_{pC}$$
$$= \frac{1}{2} m\omega^2 (A^2 - x^2) + \frac{1}{2} m\omega^2 x^2$$
$$= \frac{1}{2} m\omega^2 (A^2 - x^2 + x^2)$$
$$E_k = \frac{1}{2} m\omega^2 A^2 \quad \dots \quad \dots \quad \dots \quad (5.38)$$

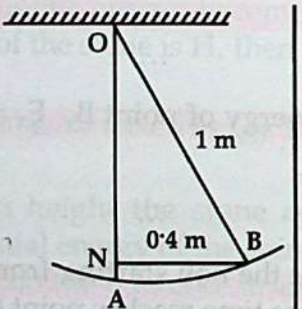
Comments : It is seen from the above equations that the total energy at point B and C is same. That means, an oscillating simple pendulum follows the 'principle of conservation of energy'.

Mathematical example

1. The mass of the bob of a simple pendulum is 0·2 kg and effective length is 1 m. If it is pulled to a distance of 0·4 m from the vertical line and is released, then find the kinetic energy and velocity of the bob while crossing the equilibrium position of motion. Analyse whether at points A and B conservation of energy is applicable or not.

Let the required velocity be v

According to the conservation principle of energy, the potential energy at the maximum point of the bob B = kinetic energy at equilibrium position A.



Here,

mass of the bob, $m = 0·2$ kg

maximum point, B = 0·4 m

equilibrium position, A = 0

$$OB^2 = ON^2 + BN^2$$

or, $ON^2 = OB^2 - BN^2$

$\therefore \quad ON = \sqrt{OB^2 - BN^2}$

Now, vertical displacement along OA is,

$$AN = OA - ON$$
$$= OA - \sqrt{OB^2 - NB^2}$$
$$= 1 - \sqrt{(1)^2 - (0·4)^2}$$
$$= 0·083 \text{ m}$$

Now, potential energy at the maximum point = mgh

According to the question,

Kinetic energy, $E_k = \frac{1}{2} mv^2 = mgh$

$$= 0·2 \times 9·8 \times 0·083 = 0·163 \text{ J}$$

Again, $\frac{1}{2} mv^2 = mgh \therefore v^2 = 2gh$

or, $v = \sqrt{2gh} = \sqrt{2 \times 9·8 \times 0·083} = 1·275 \text{ ms}^{-1}$

According to the principle of conservation of energy, potential energy at maximum point (B) from the point of suspension = kinetic energy at equilibrium point (A).

In the steady state kinetic energy $= \frac{1}{2} mv^2 = \frac{1}{2} \times 0{\cdot}2 \times (1{\cdot}275)^2 = 0{\cdot}163$ J

Again, potential energy at the highest point $= mgh = 0{\cdot}2 \times 9{\cdot}8 \times 0{\cdot}083 = 0{\cdot}163$ J
Total energy E at point A = potential energy + kinetic energy = 0 + ·163 = 0·163 J
total energy E′ at point B = potential energy + kinetic energy = 0·163 + 0 = 0·163 J
Since E = E′, hence at points A and B conservation principle of energy is applicable.

5·12 Power

When a change of motion of a body or a machine takes place due to the application of force we consider that the body or the machine has power to do work. Whether the displacement of the body is fast or slow due to the action of force cannot be understood by the amount of work, it is understood by power. Amount of work done in unit time is power.

Power is the time rate of doing work of a source and it is measured by work done in unit time.

Explanation : Suppose a person or a source does W amount of work in time t.

∴ work done in unit time or power,

$$P = \frac{\text{work}}{\text{time}} = \frac{W}{t} = \frac{Fs}{t} = Fv \quad \ldots \quad \ldots \quad \ldots \quad (5.39)$$

If the rate of doing work is not uniform, then instantaneous power,

$$P = \frac{dW}{dt}$$

If a constant force $\vec{F}$ displaces a particle by $d\vec{r}$ in time dt, then work done by th constant force,

$$dW = \vec{F} \cdot d\vec{r} \text{ and work done in unit time or power} = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

Power is a scalar quantity. Power does not depend only on the total amount o work, but depends on the time during which that work has been done. Power becomes more for doing work in short time.

For example, a machine does 2000 J of work in 4 hours. Another machine does 2400 J of work in 6 hours. Power of the first machine = 2000/4 = 500 Joule/hour. and power of the second machine = 2400/6 = 400 Joule/hour. So, although work done by the first machine is less than that of the second machine, but power of the first machine is more.

5·12·1 Unit of power

From the definition of power its unit can be found out.

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{\text{Joule}}{\text{second}} = \text{Joule/sec } (Js^{-1})$$

Dimension : $[P] = [ML^2T^{-3}]$

S. I. International unit of power is Joule/sec or watt. **Power of doing work of 1 Joule in 1 sec is called 1 Joule/sec or 1 watt.**

"Power of a machine is 50 Joule/sec"— means that it can do 50 Joule work in 1 sec.

Another unit larger than watt is also used. It is called **kilowatt (K.W.).**

Horse power : Power of doing work of 746 Joules in 1 second is called 1 horse power (H.P.).

∴ **1 H.P. = 746 J/s = 746 watt.**

PRACTICAL UNIT OF ELECTRICITY

Practical unit of power for electricity is watt. Watt is also unit of power in International system.

∴ **1 watt = 1 Js^{-1}**

∴ **1 kilowatt = 1000 watt.** That means, kilowatt is thousand times larger than watt. Recently, thousand times larger than kilowatt i.e., 1 million times larger than watt is used as unit of power. It is called **megawatt.**

$$\therefore \ \mathbf{1\ megawatt\ (MW)} = 1000 \text{ kilowatt (K.W.)} = \mathbf{10^6\ watt = 10^6\ Js^{-1}}$$

"The power of a power station is 2 megawatt". This means that energy supplied by the power station can do 2 × 10⁶ Joule or 2 mega-Joule work in 1 sec.

Dimension of power

We know,

$$\text{Power, } P = \frac{W}{t} = \frac{\text{Force} \times \text{displacement}}{\text{time}}$$

$$\therefore \text{ Dimensional equation of power [P]} = \frac{[\text{Force}]\,[\text{Displacement}]}{[\text{Time}]}$$

$$= \left[\frac{MLT^{-2} \times L}{T}\right] = [ML^2T^{-3}]$$

5·12·2 Relation between power, force and velocity

Suppose force F is acting on a body for time t. If during this time the body moves along the direction of the applied force to a distance s, then work done by that force is,

$$W = F \times s$$

$$\text{Again, power, } P = \frac{W}{t} = \frac{Fs}{t} = Fv \quad \ldots \quad \ldots \quad \ldots \quad (5.40)$$

$$\left[\because s = \frac{v}{t}\right]$$

So, power = applied force × velocity of the body.

If the displacement of the body, instead of being along the force, is along a direction making an angle θ with the applied force, then

$$P = Fv \cos\theta \quad \ldots \quad \ldots \quad \ldots \quad (5.41)$$

This equation represents a scalar product of two vector quantities.

$$\therefore \text{ According to vector notation, } P = \vec{F} \cdot \vec{v} \quad \ldots \quad \ldots \quad (5.42)$$

This equation gives the relation between power, force and velocity.

Power in case of rotational motion

In case of rotational motion we know,

work, W = torque × angular displacement

$\therefore$ Power, $P = \frac{W}{t} = \frac{\text{Torque} \times \text{angular displacement}}{\text{time}}$

$\therefore$ Power, P = Torque × angular velocity.

Mathematical example

1. How much energy is to be used in order to lift a stone of mass of 300 kg on top of the roof at a speed of 0·1 ms^{-1} by a crane ?

We know,

$P = Fv$

$\therefore P = 2940 \times 0.1$

$= 294$ W

Here,

$m = 300$ kg

$F = 300 \text{ kg} \times 9.8 \text{ ms}^{-2} = 2940$ N

$v = 0.1 \text{ ms}^{-1}$

$P = ?$

5·13 Efficiency

When we get work from a machine or a body that work is smaller than the energy supplied to that machine or body to do work. It is not only true for machine, but also true in our practical life whereat a part of work is utilised than the supplied energy. Rest of energy is lost. In case of engine this dissipation of energy is used as friction, to warm up the engine etc. This dissipation cannot be stopped completely, but by applying different technologies this dissipation can be reduced. In this case, equation of energy is equal to

supplied energy = available effective energy + used energy by other means.

Definition : The efficiency of a machine is defined as the ratio of the output energy to input or supplied energy. It is denoted by η (eta).

$\therefore$ Efficiency, $\eta = \frac{\text{output energy}}{\text{input energy}}$

Efficiency can be expressed in percentage. The unit of efficiency is HP.

Let E_1 be the energy supplied to a machine and E_2 be the dissipation of energy, then

$$\eta = \frac{E_1 - E_2}{E_1} = \left(1 - \frac{E_2}{E_1}\right) \times 100\% \quad \ldots \quad \ldots \quad (5.43)$$

No machine has 100% efficiency. For example, efficiency of a machine is 80% means that by applying 100 units of energy we can utilise 80 units of energy and the rest 20 units will be lost.

5·14 Conservative and non-conservative force

Forces are of two types; viz.—

(1) Conservative force and

(2) Non-conservative force.

5·14·1 Conservative force

The system in which mechanical energy is conserved is called conservative system and force acting in this type of system is called conservative force. In other words, if the total work done by a force in a closed path is zero, then that force is called conservative force.

Or, if a force acts on a body and that body is brought back to initial position, again by moving in any path and if work done is zero then that force is called conservative force.

Examples— Gravitational force, electric force, restoring force of an ideal spring etc.

Characteristics of conservative force :

(1) This force depends only on position.

(2) Work done by the conservative force can completely be regained.

(3) This force does not depend on the path along which the body moves from one place to another place but depends only on the initial and the final positions of the body.

(4) In the action of conservative force mechanical conservation principle of energy is followed.

(5) Total work in a complete cycle is zero.

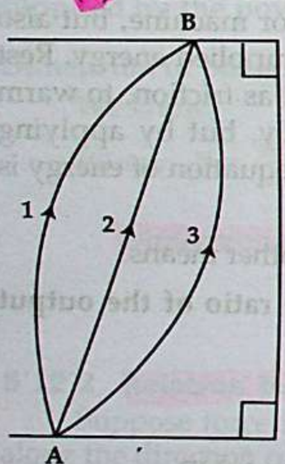


Fig. 5·24

Let an object of mass 'm' be raised from a point A to a point B whose vertical height is h [Fig. 5·24]. The transfer of the object along the paths 1, 2 and 3 is different but gravitational force mg acting at any point in any path always acts vertically downward. The displace-ment of the object along the line of action of the gravita-tional force is h. So work done along each of the three paths is same and is given by

$$W = -mgh$$

Again, if the object is transferred from point A, following path 1, to point B and again transferred from B to initial position A, thus work done in the first case is,

$$W_1 = -mgh \quad [\because h \text{ is in opposite direction of } g]$$

and in the second case,

$$W_2 = mgh \quad [\because h \text{ is in the same direction of } g]$$

$\therefore$ Total work done, $W = W_1 + W_2 = -mgh + mgh = 0$.

Since the gravitational force is conservative, so work done by this force is recoverable. According to the characteristics of conservative force, a definition of it can be given. For example, **a force is said to be conservative if the work done by the force does not depend on the path along which the body moves, but depends only on the initial and final position.**

Perceptional work : Gravitational force is a conservative force—explain.

Work done by gravitiational force depends on the initial and final position only, but not on the path of motion. Work done by this force can be regained. If a body in the gravitational field is brought to the initial position through any path, work done will be zero. So, gravitational force is a conservative force.

5·14·2 Non-conservative force

If a force acts on a body and the body is brought back to its initial position by moving in any path and if work done is not zero, then that force is called non-conservative force.

Examples— Frictional force, viscous force etc.

Or, a system in which there exists resisting force, then mechanical energy is not conserved there, rather mechanical energy is dissipated, this type of system is called non-conservative system and this resisting force is called non-conservative force.

In other words, **if total work done by an applied force in a closed path is not zero, then that force is called non-conservative force.**

Characteristics of non-conservative force :

(1) This force does not depend on the position only.

(2) Work done by this force depends on the path in transferring a body from one place to another place.

(3) Work done by the non-conservative force cannot be regained completely.

(4) Non-conservative force does not follow the principle of conservation of mechanical energy.

(5) Total work in a complete cycle is not zero.

Let an object be pushed from point A to point B along the smooth horizontal floor through path 1 [Fig. 5·25]. In this case, frictional force acts against the motion of the object. So, in this transfer work is to be done against the frictional force; because frictional force is always a force-resisting motion. Let a small segment of displacement of motion be dx and this displacement be against the frictional force F, then work done,

$$dW = -Fdx \quad \cdots \quad \cdots \quad \cdots \quad (i)$$

∴ Total work done in taking the object through path 1 from A to B is the summation of all the small work done dW, i.e.,

$$W_1 = -\int_1 Fdx.$$

Now, if the object is again taken from B to A along the path 2, the frictional force in this case also will act against the motion. So in this case also work

$$W_2 = -\int_2 Fdx.$$

In both cases since work is done against frictional force, both work will be negative and their summation will not be zero. That is, total work done,

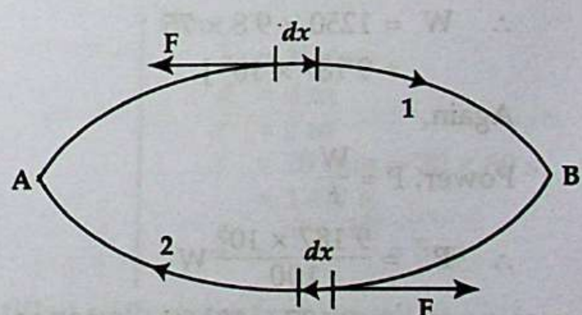


Fig. 5·25

$$W = W_1 + W_2 = -\int_1 Fdx - \int_2 Fdx \neq 0$$

So, work done by frictional force is not recoverable. Hence, frictional force is non-conservative force.

According to the characteristics of conservative and non-conservative forces it can be shown that,

to raise a body to a height h against the gravitational force F the amount of work done $= -Fh$. Now, if it is released from that height to ground, then work done will be $+Fh$.

So, in raising a body to a height h and again in bringing it back to the ground work done will be $(-Fh + Fh = 0)$ zero. Hence, gravitational force is conservative. Similarly, electric force, magnetic force etc are conservative forces.

On the other hand, in case of friction, frictional force resists the motion of a moving body. So work done by it on the body becomes negative. So, frictional force is a non-conservative force.

Perceptional work : Why frictional force is not a conservative force ? — Explain.

In this case, starting from a point and travelling through any path if it returns to the initial position then work done will not be zero. Work done by frictional force does not depend on the initial and final paths, but depends on the path of motion. Work done by frictional force cannot be regained. So, frictional force is non-conservative force.

Mathematical examples

1. A load of mass 270 kg is pulled up by a crane with a velocity of 0·1 ms^{-1}. What is the power of the crane?

We know,

$$P = \frac{W}{t} = \frac{F \times s}{t} = F \times v$$

$$= mgv \quad [\because F = mg]$$

$$\therefore P = 270 \times 9{\cdot}8 \times 0{\cdot}1 \text{ W}$$

$$= 264{\cdot}6 \text{ W}$$

Here,

Mass, $m = 270$ kg

Velocity, $v = 0.1$ ms^{-1}

Acceleration, $g = 9{\cdot}8$ ms^{-2}

Power, $P = ?$

2. A lift of mass of 900 kg ascends with a mass of 350 kg in 100 s from the ground to 18th floor to a height of 75 m. Find the work done and power applied.

We know,

Work done, $W = mgh$

$$\therefore W = 1250 \times 9{\cdot}8 \times 75$$

$$= 9{\cdot}187 \times 10^5 \text{ J}$$

Again,

$$\text{Power, } P = \frac{W}{t}$$

$$\therefore P = \frac{9{\cdot}187 \times 10^5}{100} \text{ W}$$

$$= 9{\cdot}187 \times 10^3 \text{ W} = 9{\cdot}187 \text{ kW}$$

Here,

total mass, $m = 900 + 350 = 1250$ kg

height, $h = 75$ m

times, $t = 100$ s

$g = 9{\cdot}8$ ms^{-2}

$W = ?$

$P = ?$

3. A motor of power 3430W pumps water to a height of 7·20 m from a well. If the efficiency of the motor is 90% how much water it can pump in 1 minute ? [B. B. 2006]

Let the required mass = m kg

Effective efficiency of the motor $= \eta \times P$

$$= \frac{90}{100} \times 3430 \text{ W} = 3087 \text{ W}$$

Work done per minute

$W = mgh = (m \times 9.8) \times 7.20$ J

$\therefore$ Effective power, $P = \frac{W}{t} = \frac{m \times 9.8 \times 7.20}{60}$ W

As per condition, $\frac{m \times 9.8 \times 7.20}{60} = 3087$

$\therefore$ $m = \frac{3087 \times 60}{9.8 \times 7.2} = 2625$ kg

Here,
Power, P = 3430 W
Efficiency, $\eta = 90\% = \frac{90}{100}$
Acceleration, $g = 9.8$ ms^{-2}
Height, $h = 7.20$ m
Time, $t = 1$ min $= 60$ s
Mass of water, $m = ?$

4. An engine pumps 1000 kg of water per minute from a well of depth 10 m. If 40% efficiency of the engine is lost, find the horse-power of the engine.

We know, effective power of the engine,

$P' = \frac{P \times 60}{100}$

$\therefore$ $P = \frac{P' \times 100}{60}$

Here 40% power is lost, so effective power = (100 − 40) % = 60%

$\therefore$ $P = \frac{mgh \times 100}{60 \times t} = \frac{1000 \times 9.8 \times 10 \times 100}{60 \times 60}$

$= 2.7222 \times 10^3$ watt

$= \frac{2.7222 \times 10^3}{746}$ H.P. = 3.65 H.P.

$\therefore$ P = 3.65 H.P.

Here,
$P' = \frac{mgh}{t}$
$m = 1000$ kg
$h = 10$ m
$g = 9.8$ ms^{-2}
$t = 1$ min $= 60$ s
$P = ?$

5. Depth and diameter of a well full of water are 10 m and 4 m respectively. A pump can make the well empty in 20 minutes. Calculate the horse power of the pump.

We know,

$P = \frac{W}{t} = \frac{mgh}{t} = \frac{V\rho gh}{t} = \frac{\pi r^2 l \rho gh}{t}$

$= \frac{3.14 \times (2)^2 \times 10 \times 10^3 \times 9.8 \times 5}{1200}$

$= 5128.67$ W $= \frac{5128.67}{746}$ H.P.

$= 6.87$ H.P.

Here,
$l = 10$ m
$d = 4$ m
$r = 2$ m
$t = 20$ min $= 20 \times 60$ s $= 1200$ s
$h = \frac{0 + 10}{2} = 5$ m

6. The rope of a pulley can lift a bucket filled with water from a well at uniform speed of 0.70 ms^{-1}. If the rope applies power of 20 kW, then what will be the tension on the rope ?

We know,

$P = Fv$

or, $F = \frac{P}{v}$

$= \frac{20 \times 10^3}{0.70}$

$= 28.57 \times 10^3$ N

Here,
$v = 0.70$ ms^{-1}
$P = 20$ kW $= 20 \times 10^3$ W
tension, $F = ?$

7. Depth and diameter of a well filled with water are respectively 10 m and 1˙5 m. A pump can make the well empty in 20 minutes. Calculate the horse-power of the pump. If another pump of power 0˙4 H.P. is connected, then how much time will be saved ? [J. B. 2015]

Here,

depth of the well, $l = 10$ m

diameter of the well, $d = 1.5$ m

radius of the well, $r = 0.75$ m

time, $t = 25$ min $= 25 \times 60 = 1500$ s

average height, $h = \frac{0 + 10}{2} = 5$ m

power, P = ?

We know,

$$P = \frac{W}{t} = \frac{F \times h}{t}$$

$$= \frac{mgh}{t} \quad [\because m = v\rho = \pi r^2 l\rho]$$

$$= \frac{\pi r^2 l\rho gh}{t}$$

$$= \frac{3.14 \times (0.75)^2 \times 10 \times 10^3 \times 9.8 \times 5}{1500}$$

$$= 576.975 \text{ W} = \frac{576.975}{746} \text{ H.P.}$$

$$= 0.773 \text{ H.P.}$$

$\therefore$ total power, $P + P_1 = 0.773 + 0.4 = 1.173$ H.P.

If time taken to empty the well by the two pumps combined is t, then

$$P + P_1 = \frac{W}{t}$$

$$\text{or, } t = \frac{W}{P + P_1} = \frac{\pi r^2 l\rho gh}{1.73}$$

$$= \frac{3.14 \times (0.75)^2 \times 10 \times 10^3 \times 9.8 \times 5}{1.173}$$

$$= 989.0345 \text{ s} = 16.48 \text{ min.}$$

$\therefore$ Time saved = (25 — 16˙48) min = 8˙52 min = 8 min 31 sec

Necessary mathematical formulae

work, $W = \vec{F} \cdot \vec{s}$ ... ... (1)

work, $W = Fs \cos\theta$ ... ... (2)

work done in stretching a spring, $W = \frac{1}{2}Kx^2$ ... ... (3)

spring constant, $K = \frac{F}{x}$ ... ... (4)

potential energy, $E_p = mgh$ ... ... (5)

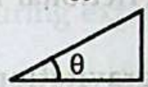
kinetic energy, $E_k = \frac{1}{2}mv^2$ ... ... (6)

power, $P = \frac{W}{t} = \frac{mgh}{t} = Fv$ ... ... (7)

elastic potential energy, $E_p = \frac{1}{2}Kx^2$ ... ... (8)

mechanical energy, $E = E_p + E_k$ ... ... (9)

efficiency, $\eta = \dfrac{\text{effective energy}}{\text{total supplied energy}}$ ... ... (10)

power, $P = \vec{F}.\vec{v} = Fv \sin \theta$  ... ... (11)

effective power, $P' = \eta \times$ actual power (P) ... ... (12)

work, $W = \frac{1}{2} mv^2 - \frac{1}{2} mv_0^2$ ... ... (13)

$W = \Delta K$ ... ... (14)

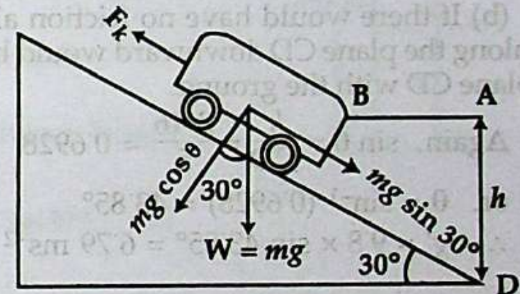
Higher efficiency mathematical example

1. A driver was driving a truck of 1000 kg weight through an inclined road of 30° angle. The speed of the truck was 20 ms^{-1}. All on a sudden by seeing a boy 50 m distance in front the truck was stopped.

(a) How much above the ground the truck is ?

(b) Will the conservation principle be applicable in this case ?— Explain. [Say, frictional force = 11150 N]

(a) Suppose the truck was stopped at point D travelling 50 m from point B. So, vertical distance between B and D is AD = h



$\therefore \sin 30° = \frac{h}{50}$ or, $\frac{1}{2} = \frac{h}{50}$

or, $h = \frac{50}{2} = 25$ m

At point B the truck is 25 m above the ground.

(b) Total energy at

point B = kinetic energy + potential energy

$= \frac{1}{2} mv_B^2 + mgh$

$= \frac{1}{2} \times 1000 \times (25)^2 + 1000 \times 9.8 \times 25$

$= 500 \times (25)^2 + 25000 \times 9.8$

$= 312500 + 245000 = 557500$ J

Here,
$m = 1000$ kg
$\theta = 30°$
$s = 50$ m
Final velocity, $v = 0$
Initial velocity, $v_0 = 20\ ms^{-1}$
Frictional force = 11150 N

Velocity at point D = 0, kinetic energy = 0, potential energy = 0

$\therefore$ Transformation of energy due to friction

= necessary energy to stop the truck at point D

= Frictional force × displacement $= F_k \times s = 11150 \times 50 = 557500$ J

$\therefore$ Total energy of the truck at point D = 557500 J + 0 + 0 = 557500 J

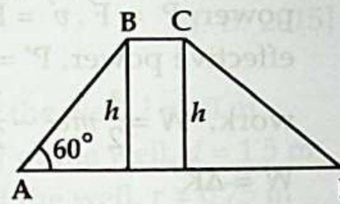
$\therefore$ Total energy at point B = total energy at point D.

So, the truck follows the conservation principle.

2. A boy of mass of 30 kg ascends in a ladder AB as shown in the figure and descends along the inclined plane CD. Frictional force of the plane is 50 N. In the figure AB = 4 m, BC = 1 m and CD = 5 m [Ch. B. 2015]

(a) Calculate the work done by gravitational force in order to reach the boy from A to point C.

(b) Whether the acceleration due to gravity of the boy while descending along the path CD is greater or smaller than acceleration due to gravity ? Analyse mathematically.



(a) If h is the height of plane BC with respect to AD, then $\frac{h}{AB} = \sin 60°$

$\therefore \quad h = AB \sin 60° = 4 \times \frac{\sqrt{3}}{2} = 3.46$ m

work done in going from B to point C, $W = mg \times BC = m \times 0 \times BC = 0$

$\therefore$ work done against gravitational force to reach from A to point C,

$W = E_p = 30 \times 9.8 \times 3.46 = 1018.4$ J

(b) If there would have no friction along the path CD, then the acceleration of the boy along the plane CD downward would have been, $g' = g \sin\theta$; here θ is the inclination of the plane CD with the ground.

Again, $\sin\theta = \frac{h}{CD} = \frac{3.46}{5} = 0.6928$

or, $\theta = \sin^{-1}(0.6928) = 43.85°$

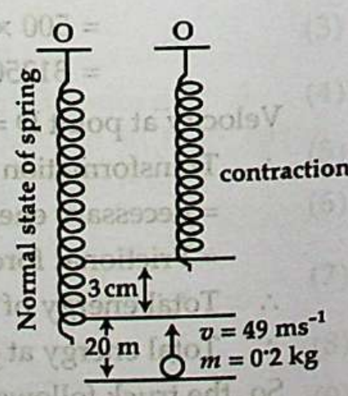
$\therefore \quad g' = 9.8 \times \sin 43.85° = 6.79$ ms^{-2} [law : acceleration due to gravity along any inclined plane is $g' = g \times \sin\theta$]

$\therefore \quad g' < g$. So if there would have no friction then acceleration along CD downward would have been 6.79 ms^{-2}, and if there is friction acceleration will be further less. Hence, acceleration of the boy descending along the path CD is smaller than acceleration due to gravity.

3. One end of a spring is hung from point O as shown in the figure. A ball of mass of 0.2 kg is thrown with velocity 49 ms^{-1}, it strikes the other end of the spring at 20 m above and contracts the spring by 3 cm, the spring also applies restoring force on the ball. [R. B. 2015]

(a) Calculate the velocity of the ball just before touching the ground.

(b) Is it possible to determine the work done by the spring force from the stimulus ? Explain with mathematical reasoning.



(a) Magnitude of velocity of the ball just before touching the ground will be equal to the initial velocity, but the direction will be reverse i.e., magnitude of velocity will be 49 ms^{-1}. This is because, that after throwing and return to the ground of the ball both gravitational force and spring force acting on it is conserved and if it returns to the original position by completeing one complete cycle, then work done by the conservative force is zero.

(b) Work done by the spring force is zero. This is because that velocity of the ball while touching the spring and the velocity just after release from the spring will have same velocity. Work done by spring force during contraction of the spring will be negative work and work done during expansion will have equal positive value; as a result total work done will be zero.

work done by the spring force = kinetic energy of the ball at the instant of striking

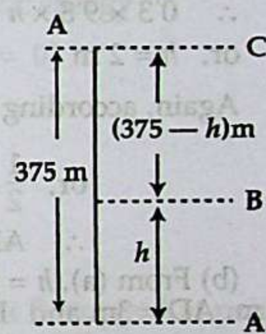
$$= \frac{1}{2} mv^2 = \frac{1}{2} \times 0.2 \times 2000 = 200.9 \text{ J}$$

So, work done by the spring force of the stimulus can be determined.

4. Height of the top of the Petronus twin tower is 375 m. Kashem ascends on the top with a body of mass of 10 kg and time taken for this is 40 minutes. He dropped the body from the top. It reached the ground without any resistance. Monir said, 'I can do this work.' Mass of Kashem and mass of Monir are respectively 60 kg and 45 kg. **[S. B. 2015]**

(a) At what height from the ground potential energy will be double than the kinetic energy ?

(b) Will Monir be able to do the work at the same time ? Give your opinion with mathematical analysis.



(a) Suppose at the height of h from the ground potential energy is double than kinetic energy.

Potential energy at a height h from the ground,

$$E_p = mgh \quad \ldots \quad \ldots \quad \text{(i)}$$

Since the height of the tower is 375 m, hence height of the point B from the top = $(375 - h)$ m.

kinetic energy, $E_k = \frac{1}{2} mv^2 = \frac{1}{2} m \, [v_0^2 + 2g(375 - h)]$

$= \frac{1}{2} m \, [2g(375 - h)]$

$= mg \, (375 - h) \quad \ldots \quad \ldots \quad \text{(ii)}$

According to question, $E_p = 2E_k$

or, $mgh = 2 \times mg \, (375 - h)$

or, $h = 2 \, (375 - h)$

or, $h = 750 - 2h$

or, $3h = 750$

$\therefore \; h = 250$ m

(b) According to the stimulus, height of the tower, $h = 375$ m

In case of Kashem, mass $m = (10 + 60)$ kg = 70 kg

time, $t = 40$ min = 40×60 sec = 2400 sec

$\therefore$ power, $P = \frac{W}{t} = \frac{mgh}{t} = \frac{70 \times 9.8 \times 375}{2400} = 107.2$ watt

Again, in case of Monir, mass $m' = (10 + 55)$ kg = 65 kg

time, $t = 40$ min = 2400 sec

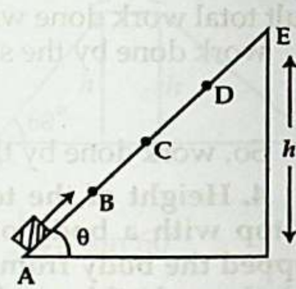
power, $P' = \frac{W'}{t} = \frac{m'gh}{t} = \frac{65 \times 9.8 \times 375}{2400} = 99.5$ watt

If Monir applies power of 99.5 watt, he can do the work.

5. If a body of mass 300 g is placed horizontally at an angle of 30°, and if 5·88 J kinetic energy is applied it can reach without friction from A to point E. Just a moment later the body started falling from the point E along above plane to A. According to the figure AB = BC = CD = DE.

(a) Calculate the length of the inclined plane AE.

(b) The body obeys the conservation principle of mechanical energy while falling through that plane— evaluate its justification by mathematical analysis at points D and C.



(a) mass, m = 300 g = 0·3 kg; mid angle, θ = 30°; kinetic energy, E_k = 5·88 J

If the height of the inclined plane is h, then

$W = mgh = E_p$

$\therefore\ 0{\cdot}3 \times 9{\cdot}8 \times h = 5{\cdot}88$

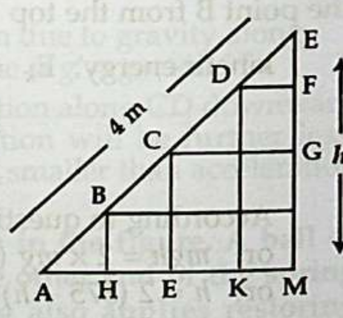
or, $h = 2$ m

Again, according to the figure, $\sin 30° = \frac{h}{AE}$

or, $\frac{1}{2} = \frac{2}{AE}$

$\therefore\ AE = 4$m

(b) From (a), h = 2m, AE = 4m. Again, since AB = BC = CD = DE hence AC = EC = 2 m, AD = 3m and ED = 1m



We get, $\sin A = \frac{h}{AE}$

Potential energy at point D, $E_p = mg \times DK$ ... (i)

kinetic energy at point D,

$E_k = \frac{1}{2} mv^2 = \frac{1}{2} m\ [v_0^2 + 2g\ (EF)]$

$= \frac{1}{2} m\ [2g\ (EM - DK)]\ \because v_0 = 0$ ... (ii)

$\therefore$ total energy at point D $= E_p + E_k$

$= mgDK + mgEM - mgDK$

$= mgEM$

Potential energy at point C, $E_p = mg\ E'C$ ... (iii)

kinetic energy at point C, $E_k = \frac{1}{2} mv^2 = \frac{1}{2} m\ (2g\ E'G)$

$= \frac{1}{2} m \times 2g\ (EM - E'C)$

$= mg\ EM - mgE'C$ ... (iv)

$\therefore$ total energy at point C, $= E_p + E_k = mgE'C + mgEM - mgE'C = mgEM$

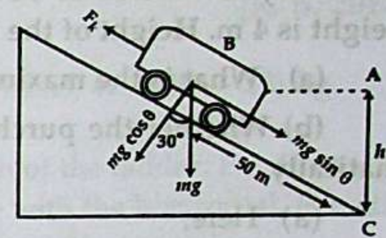
It is seen that at points D and C total energies are same.

So, at points D and C of the inclined plane conservation principle of mechanical energy is obeyed.

6. Shafiq is moving in a car of mass 1500 kg in a hilly road which is inclined at an angle of 30° with the horizontal. The velocity of the car is 25 ms^{-1}. Seeing a car ahead the car stopped by travelling 50 m distance.

(a) What is the frictional force acting on the car ?

(b) Analyse whether in this case the car obeys the conservation principle of energy or not.



(a) Given, mass of the car, m = 1500 kg

displacement, s = 50 m

final velocity, v = 0

initial velocity, v_0 = 25 ms^{-1}

Let the resisting force be F_k

Work done by the net force = change of kinetic energy of the body

or, force × displacement = initial kinetic energy − final kinetic energy

or, $(F_k - mg \sin 30°) \times 50 = \frac{1}{2} m v_0^2 - \frac{1}{2} m v^2 = \frac{1}{2} m v_0^2 \quad [\because v = 0]$

or, $\left(F_k - 1500 \times 9{\cdot}8 \times \frac{1}{2}\right) \times 50 = \frac{1}{2} \times 1500 \times (25)^2$

or, $Fk = \frac{1500 \times (25)^2}{2 \times 50} + \frac{1500 \times 9{\cdot}8}{2}$

$\therefore$ F_k = 16725 N

(b) Again, suppose the car after travelling 50 m from point B stopped at point C. According to the figure vertical distance between points B and C is AC = h

$\therefore \quad \sin 30° = \frac{h}{50}$

or, $h = 50 \times \frac{1}{2} = 25$ m

total energy of the car at point B = kinetic energy + potential energy

$= \frac{1}{2} m v^2 + mgh$

$= \frac{1}{2} \times 1500 \times (25)^2 + 1500 \times 9{\cdot}8 \times 25 = 836250$ J

velocity of the car at point C = 0

kinetic energy = 0

height of the car from the ground, $h = 0$

potential energy = 0

transformation of energy due to frictional force = necessary energy to stop the car

= frictional force × displacement

$= F_k \times s$ = (16725 × 50)

= 836250 J

Total energy at point C = potential energy + kinetic energy + transformation energy due to friction

= 0 + 0 + 836250 = 836250 J

So, total energy at point B and C is same. So, the car obeys conservation principle of energy at points B and C.

7. Mr. Sohel bought a pump of 1·2 kW power to raise water from the underground reserve tank to the roof of the building. On the surface of the pump efficiency is written as 90%. The tank is cylindrical and its diameter is 2 m and height is 4 m. Height of the roof from the tank is 28 m.

(a) What is the maximum work the pump can do in a day ?

(b) Whether the purchase of the pump was correct or not ? — analyse mathematically.

(a) Here,

electric power expended by the pump, P = 1·2 kW

efficiency, $\eta = 90\% = 0·9$

So, effective power P′ = efficiency × actual power

$P' = 0·9 \times 1·2 \text{ kW} = 1·08 \text{ kW}$

1 day = 24 × 60 × 60 = 86400 sec

The pump can do maximum work in a day,

$$W = P \times t = 1·08 \text{ kW} \times 86400 \text{ sec}$$
$$= 1·08 \times 10^3 \text{ W} \times 86000 \text{ sec}$$
$$= 93·3 \times 10^6 \text{ J}$$

(b) diameter of the cylindrical shape underground tank, $d = 2$ m, height, $h = 4$ m

∴ internal volume, $V = \frac{1}{4}\pi r^2 h = \frac{1}{4} \times 3·14 \times (2)^2 \times 4 = 12·56 \text{ m}^3$

If the tank is completely filled with water, then the mass of water,

$$m = V\rho = 12·56 \times 1000 = 12560 \text{ kg}$$

Average height of raised water, $h = \left(28 + \frac{4}{2}\right) \text{m} = 30 \text{ m}$

∴ applied power per hour by the pump, $P = \frac{W}{t} = \frac{mgh}{t}$

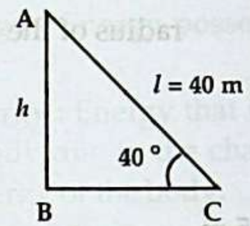
$$= \frac{12560 \times 9·8 \times 30}{3600}$$
$$= 1025·8 \text{ W}$$
$$= 1·0258 \text{ kW}$$

So, necessary power of the pump is 1·0258 kW which is less than the power of the pump purchased by Mr. Sohel. The pump can raise the total amount of water from the tank in less than 1 hour; so it was right to buy the pump.

8. A person of mass of 80 kg ascended on the roof of a building carrying 20 kg mass on his hand along a ladder of length of 40 m. The ladder was attached to the building making an angle 40° with the horizontal.

(a) Calculate the work done by the person.

(b) If the ladder is made 60 m, then at what angle it should be placed so that same amount of work will be done and in that case whether any benefit will be found or not.—Give opinion mathematically. [R. B. 2017]



Here,
total mass, $m = (80 + 20)$ kg
$= 100$ kg
length of the ladder, $l = 40$ m
angle with the horizontal, $\theta = 40°$
height of the roof $= h$
work done, W = ?

(a) We know,

$\sin\theta = \frac{h}{l}$

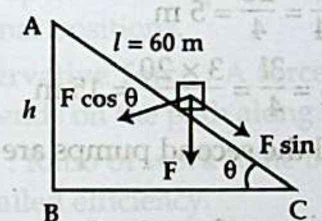
$\therefore\ h = l \times \sin\theta = 40 \sin 40° = 25{\cdot}71$ m

and work done, $W = mgh$

$= 100 \times 9{\cdot}8 \times 25{\cdot}71$

$= 25195{\cdot}85$

(b)



Here,
total mass, $m = 100$ kg
length of the ladder, $l = 60$ m
height of the roof, $h = 25{\cdot}71$ m

Since, in both cases height of the roof is same, so same amount of work will be done.

Suppose the ladder is placed at an angle of θ with the horizontal.

$\therefore$ height of the roof, $h = l \sin\theta = 60 \times \sin\theta$

$\therefore\ 25{\cdot}71 = 60 \times \sin\theta$

or, $\sin = \frac{25{\cdot}71}{60} = 0{\cdot}4285$

$\therefore\ \theta = \sin^{-1}(0{\cdot}4285) = 25{\cdot}37°$

Here, same amount of work will be done if the ladder is placed at an angle of 25·37° with the horizontal. According to the figure as the value of θ decreases, the value of F sin θ will be less and difficulty will be less to rise above.

Since the value θ has been reduced than the previous value, so here less effect will be needed for the person to rise above.

9. Depth of a well filled with water is 20 m and its diameter is 2 m. A pump of 5 HP is attached in order to make the well empty. After removing half of water from the well the pump was damaged. To remove the rest amount of water another pump of same power is added.

(a) Calculate the work done by the first pump.

(b) Wheter same time will be needed to remove water by the first and second pump or not.—Show by mathematical analysis. [Ch. B. 2017]

Here,
depth of the well, $l = 20$ m
radius of the well, $r = \frac{2}{2}$ m $= 1$ m

(a) In the first case, volume of water removed,

$$V = \frac{\pi r^2 l}{2} = \frac{3.1416 \times (1)^2 \times 20}{2}$$
$$= 31.416 \text{ m}^3$$

Mass of the removed water,

$$m = V\rho = 31.416 \times 1000 = 3.1416 \times 10^4 \text{ kg}$$

Average displacement of water, $h_1 = \frac{l}{4} = \frac{20}{4} = 5$ m

Work done by the first pump,

$$W = mgh$$
$$= 31.416 \times 10^3 \times 9.8 \times 5$$
$$= 1.54 \times 10^6 \text{ J}$$

(b) Power of both the pump, P $= 5$ HP $= 5 \times 746 = 3730$ watt

mass of water in both cases, $m = 3.1416 \times 10^4$ kg

In the first case, average displacement, $h_1 = \frac{l}{4} = \frac{20}{4} = 5$ m

In the second case, average displacement, $h_2 = \frac{3l}{4} = \frac{3 \times 20}{4} = 15$ m

If time taken to remove water by the first and the second pumps are respectively, t_1 and t_2, then

in the first case, $P = \frac{W_1}{t_1}$ $\therefore t_1 = \frac{mgh_1}{P} = \frac{31.416 \times 10^3 \times 9.8 \times 5}{3730} = 412.70$ sec

in the second case, $P = \frac{W_2}{t_2}$ $\therefore t_2 = \frac{mgh_2}{P} = \frac{31.416 \times 10^3 \times 9.8 \times 15}{3730} = 1238.11$ sec

Mathematically, it is seen that $t_1 < t_2$. So time taken by the first and the second pumps will not be same. It will take more time by the second pump to remove water.

Summary

Work : Product of the force and the component of displacement along the direction of the force is called work.

Unit of work : Unit of work is Newton-metre or Joule to do work. It is measured by the amount of work a body can do.

Energy : Capacity or ability to do work is called energy.

Elastic force : Within elastic limit, due to the application of external force if a body is changed in size and shape and after removal of the force if the body regains its original shape, then that force is called elastic force.

Positive work : Work done by a force is called positive work.

Negative work : Work done against a force is called negative work.

Workless force : Force acting perpendicular to the displacement of a body does not do any work during displacement. This type of force is called workless force.

Force due to gravity : The attractive force between the earth and a body above or near the surface of the earth is called force due to gravity.

Kinetic energy : Energy possessed by an object by virtue of its motion is called kinetic energy.

Potential energy : Energy that is acquired due to the position of a body or, energy acquired by the body due to the change of position of the particles within the body is called potential energy of the body.

Power : Power is the time rate of doing work. It is measured by the work done in unit time.

Unit of power : Unit of power is Joules/sec (Js^{-1}).

1 watt : Power of doing work in one second is called 1 watt.

1 horse-power : Power of doing 746 Joules of work in 1 second is called 1 horse-power.

Conservative force : A force is said to be conservative if the work done by the force does not depend on the path along which the body moves, but depends only on the initial and final positions.

Non-conservative force : A force is said to be non-conservative if the work done by the force depends on the path along which the body moves.

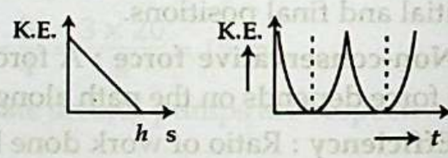
Efficiency : Ratio of work done by a machine to the amount of energy supplied to the machine is called efficiency.

Conservation principle of mechanical energy : Energy can neither be created nor destroyed but can only be converted from one form to another. Kinetic energy of a body is transformed into potential energy and potential energy is transformed into kinetic energy only. Total energy of the body remains always constant. This is called the conservation principle of mechanical energy.

Summary of the relevant topics for the answer of multiple choice questions

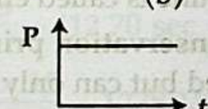
1. It is difficult to ascend stairs because here work is done against the gravitational force. Gravitational unit of work is kg-metre.
2. If the momentum of a moving body is P and kinetic energy is K then relation between then : $K = \frac{\vec{P}\cdot\vec{P}}{2m}$ or, $\frac{P^2}{2m}$. Work done by a body rotating in circular path is zero. If a body is raised upward, power of the machine, $P = F \times v = mgv$. Maximum value of potential energy is at infinity and its maximum value is zero.
3. Electric energy is transformed into light energy by an electric bulb. Opposite work of gravitational force, $W \propto h$.

4. If both mass and velocity of a body is made double, then kinetic energy increases 4 times. Work done by centripetal force is zero.
5. If a spring is contracted, then potential energy is stored in it. Work done against elastic force, $W \propto x^2$.
6. Power, $P = \frac{W}{t} = \frac{Fs}{t} = Fv = mgv$. Dimension of kinetic energy is $[ML^2T^{-2}]$. In case of positive work kinetic energy increases and acceleration is produced.
7. In case of conservative force—
   (1) total work done in a complete cycle is zero.
   (2) amount of work does not depend on the path of motion.
   (3) conservation of energy is obeyed.
   (4) work can be regained.
   Examples —electric force, gravitational force, restoring force of a spring.
8. In case of non-conservative force—
   (1) total work in a complete cycle is not zero,
   (2) amount of work depends on the path of motion,
   (3) conservation of energy is not obeyed
   (4) work cannot be recovered completely.
   Examples—frictional force, viscous force.
9. A body is thrown upward from the ground and after ascending the height h it returns to the ground. In the adjoining figure (a) it is shown. In fig. (b) maximum and minimum kinetic energies are shown.

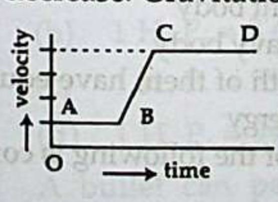


(a) (b)

10. A body at rest starts moving as a constant force acts on it. If friction is not considered, then adjoining figure shows the power of the body. Work done by centripetal force is zero.



11. If the magnitude of momentum becomes equal to the value of kinetic energy then velocity of the body is 2 ms^{-1}.
12. In ascending through a stair work done is negative and in descending work done is positive. Dimension of energy is $[ML^2T^{-2}]$.
13. If angle between force and displacement is 0, then work becomes maximum and for 90° it is minimum.
14. Dimensional equation of power is $[ML^2T^{-3}]$. Potential energy of a gas of mass m inside a cube of height h is zero.
15. The ratio of linear momenta of two bodies A and B having masses 9 g and 4 g respectively and equal kinetic energies is 3 : 2.
16. If the momentum of a body is increased by 100%, its kinetic energy is increased by 300%.
17. The ratio of effective energy and supplied energy of a machine is called efficiency.
18. If kinetic energy is increased 4 times, then momentum increases 2 times. In positive work, kinetic energy increases and acceleration is produced.

19. If angle between force and displacement is θ, then condition for negative work is $180° \geq \theta \geq 90°$.
20. Work done by a force or condition of positive work is, $0° \leq \theta < 90°$. Gravitational unit of work is kg-metre.
21. Value of work will be minimum or zero if angle between froce and displacement is 90°.
22. Due to change of shape of a body potential energy is acquired—when a bow is pulled by placing an arrow, a metal plate is curved.
23. Examples of work done by variable forces— (i) work done in gravitational field, (ii) work done by electric force.
24. Conditions of zero work— (i) $\cos\theta = 0$, (ii) if no displacement occurs even due to application of force on a body.
25. Potential energy of a body depends on mass and height. Force constant or spring constant, $K = \frac{F}{x}$. Dimension, MT^{-2}.
26. Keeping on head a heavy body is shifted from one place to another place on a road along horizontal direction— (1) work is done against a frictional force (2) work done is zero by a normal reaction.
27. If distance between two particles is increased (i) work done by gravitational force is zero, (ii) work done by external force is positive, (iii) work done by gravitational force depends on initial and final values, not on the angle between them. Relatior between potential (V) and intensity (E) is, $E = -\frac{dV}{dr}$.
28. Equation for expressing work and potential energy in case of contraction and expansion of a spring is, $W = \frac{1}{2}Kx^2$. That means, work done by elastic force is proportional to the square of displacement.
29. If a bomb thrown from an aeroplane is burst in air then total momentum will decrease. Gravitational potential energy is proportional to displacement.
30. 

(i) According to the figure, momentum of the portion CD will be four times the momentum of the portion AB.
(ii) if velocity of the portion CD is double, then kinetic energy will be four times of portion AB.

EXERCISE

(A) Multiple choice questions

1. If applied force and displacement are oppositely directed, then what will be the work done ?
   (a) positive
   (b) negative
   (c) zero
   (d) maximum

2. How will be the work done if the angle between applied force and displacement is zero ?
   (a) positive
   (b) negative
   (c) zero
   (d) minimum